AREAL REDUCTION FACTORS FOR DESIGN RAINFALL ESTIMATION IN THE C5 SECONDARY DRAINAGE REGION OF SOUTH AFRICA

by

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DECLARATION

I, the undersigned, declare that the dissertation hereby submitted by me for the degree *Magister Technologiae* (Engineering: Civil) at the Central University of Technology, Free State, is my own independent work and has not been submitted by me to another University and/or Faculty in order to obtain a degree. I further cede copyright of this dissertation in favour of the Central University of Technology, Free State.

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ABSTRACT

Design point rainfall estimates assume a uniform distribution of rainfall over a catchment and hence are only representative of a limited area. For larger areas, Areal Reduction Factors (ARFs) are used to convert design point rainfall depths or intensities to an average areal design rainfall depth or intensity for a catchment-specific critical storm duration and catchment area. The overall purpose of this study is to develop an enhanced methodology to express the spatial and temporal rainfall variability at a Quaternary Catchment (QC) level by means of geographically-centred and probabilistically correct ARFs. The ARF values presented in this study are based on observed daily rainfall data as extracted from 223 rainfall stations situated in the C5 secondary drainage region. The methodology adopted is based on a modified version of Bell's (1976) geographically-centred approach. Individual sets of ARF values were derived for each of the 23 QCs present in the C5 secondary drainage region by considering various storm durations (1, 8, 16, 24, 72 and 168 hours) and corresponding recurrence intervals (2, 5, 10, 20, 50, 100 and 200 years). The climatological variability in the two tertiary drainage regions (C51 and C52) of the C5 was also recognised by conducting separate regression analyses in each region. The statistical differences in the regional ARF values highlight the presence of dominant weather types in each region. The statistical differences also confirm that ARFs are influenced by different rainfall-producing mechanisms while not being constant for various storm durations and exceedance probabilities or recurrence intervals such as geographically-centred probabilistically correct ARFs. It is recommended that the findings from this study be expanded to other regions in South Africa, ultimately to devise both improved design rainfall and flood estimates.

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LIST OF ABBREVIATIONS

AEP	Annual Exceedance Probability
AMS	Annual Maximum Series
ARF	Areal Reduction Factor
ARFT	Areal Reduction Factor Tool
CAPPI	Constant Altitude Plan Position Indicator
CSIR	Council for Scientific and Industrial Research
DCR	Daily Catchment Rainfall
DDF	Depth-Duration-Frequency
DREU	Daily Rainfall Extraction Utility
DWAF	Department of Water Affairs and Forestry
DWS	Department of Water and Sanitation
EMA	Expectation Maximisation Algorithm
ES	Escarpment
ESRI	Environmental Systems Research Institute
EV1	Extreme Value Type I
GEV/LM	General Extreme Value using Linear Moments
GEV/PWM	General Extreme Value using Probability Weighted Moments
GIS	Geographical Information Systems
GLO	Generalised Logistic
GLO/LM	Generalised Logistic using Linear Moments
GOF	Goodness-of-Fit
HI	Highveld
HRU	Hydrological Research Unit
IDW	Inverse Distance Weighted
ISCW	Institute for Soil, Climate and Water
KAL	Kalahari Desert
KAR	Karoo
LEV1	Log-Extreme Value Type I
LM	Linear Moments
LN	Log-Normal
LO	Lowveld
LP3/MM	Log-Pearson Type III using Method of Moments

MAE	Mean Annual Evaporation
MAP	Mean Annual Precipitation
MED	Mediterranean
MI	Monthly Infilling
MM	Method of Moments
MML	Method of Maximum Likelihood
MR	Median Ratio
MRC	Modder River Catchment
MSR	Maximum Station Rainfall
MSR_{CS}	Maximum Station Rainfall – Catchment Statistic
MSR_{SS}	Maximum Station Rainfall – Single Station Statistic
NERC	Natural Environment Research Council
NAM	Namib Desert
NW	North-west Cape
PDS	Partial Duration Series
PWM	Probability Weighted Moments
QC	Quaternary Catchment
RI	Recurrence Interval
RL	Record Length
RLMA	Regional L-Moment Algorithm
RLMA&SI	Regional L-Moment Algorithm and Scale Invariance
RRC	Riet River Catchment
SANRAL	South African National Roads Agency
SASEX	South African Sugar Association Experiment Station
SAWB	South African Weather Bureau
SAWS	South African Weather Services
SC	Southern Coastal
SI	Scale Invariance
SRR	Smithers Regional Rainfall
SRTM	Shuttle Radar Topography Mission
TR	Technical Report
UK FSR	United Kingdom Flood Studies Report
USGS	United States Geological Survey
USWB	United States Weather Bureau

CHAPTER 1 : INTRODUCTION

This chapter provides some background on the spatial and temporal analysis of rainfall distribution using Areal Reduction Factors (ARFs) and includes the problem statement and purpose of the study. The layout of the dissertation structure is provided at the end.

1.1 Background

Rainfall in South Africa may occur in many different ways, such as convective, orographic and/or frontal rainfall, as well as tropical cyclones occurring irregularly in the north-eastern parts of the country (Haarhoff and Cassa, 2009; Van der Spuy and Rademeyer, 2014). Flood-producing rainfall events are also characterised by an uneven spatial and temporal distribution. Typically, rainfall storms could have one or more maximum rainfall cores, while displaying for any given period a sensibly smooth non-linear reduction in average areal values with an increasing distance from the maximum rainfall core (Alexander, 2001).

The practising engineer or hydrologist is mainly concerned with design rainfall, such as rainfall information derived from observed rainfall data and which comprised of a depth and duration associated with a given Recurrence Interval (RI or *T*) or Annual Exceedance Probability (AEP) (Gericke and Du Plessis, 2011). However, design point rainfall estimates are only representative of a limited area. For larger areas, the areal average design rainfall depth or intensity is likely to be less than the maximum design point rainfall depths or intensities (Siriwardena and Weinmann, 1996). ARFs are used to describe this relationship between point and areal rainfall, in other words, ARFs are used to convert design point rainfall depths or intensities to an average areal design rainfall depth or intensity for a catchment-specific critical storm duration and catchment area (Alexander, 2001).

In many countries, the current ARF approaches are mostly based on empirical methods, whether geographically-centred or storm-centred. According to Asquith and Famiglietti (2000), storm-centred approaches have not seen widespread application due to the difficulty of including multi-centred storms.

Omolayo (1993) indicates that storm-centred approaches are not suitable for estimating design areal rainfall from design point rainfalls since extreme design point rainfall and extreme areal design rainfall are unlikely to be produced by the same rainfall event or rainfall type. Omolayo (1993) also suggests that to obtain a probabilistically correct ARF for a critical storm duration, the *T*-year regional areal design rainfall should be divided by the weighted average *T*-year design point rainfall of all the rainfall stations in the same region. In other words, a geographically-centred approach should be used.

The first attempt in South Africa to analyse ARFs based on a storm-centred approach was by Van Wyk (1965, cited by Lambourne and Stephenson, 1986) on a small-scale (catchment areas ≤ 800 km^2) in Pretoria, Gauteng, South Africa. In addition, a few rainfall storm areas from the United States of America (USA) and Canada were also analysed for comparison purposes. In the late 1960s, Wiederhold (1969, cited by Lambourne and Stephenson, 1986) used a variable location, storm-centred approach, *i.e*. a modified version of Van Wyk's (1965) method, to establish ARFs for catchment areas between 500 km^2 and 30 000 km^2 within 18 regions delineated for South Africa. These approaches posed conceptual problems when applied to a geographically-centred catchment and the use of a correction factor was suggested. In response, Alexander (1980) developed a geographically-centred ARF relationship based on the ARFs contained in the United Kingdom Flood Studies Report (UK FSR; NERC, 1975). This geographically-centred ARF relationship is expressed as a function of the catchment area and catchment response time in terms of the time of concentration (T_C) and revealed slightly more conservative results when compared to the UK FSR and the United States Weather Bureau (USWB) ARF values (Alexander, 2001).

During the last three decades, several new analytical methods have been proposed to estimate ARFs such as the storm movement (Bengtsson and Niemczynowicz, 1986), crossing properties (Bacchi and Ranzi, 1996), spatial correlation structure (Sivapalan and Blöschl, 1998) and scaling relationships (De Michéle, *et al*., 2001). In recent years, radar rainfall information has also become more readily available in many parts of the world and assists in improving spatial and temporal resolutions. The use of radar rainfall information requires considerably less computational effort and data compared to the empirical methods (Svensson and Jones, 2010).

The research question, which focuses on problems associated with the temporal and spatial distribution of rainfall when the relationships between point and areal design rainfall are estimated, is discussed in the next section.

1.2 Problem Statement

The empirical and/or analytical estimation of ARFs on a large scale is limited to the UK (UK FSR, 1975), USA (USWB, 1957; 1958) and Australia (Siriwardena and Weinmann, 1996). Omolayo (1993) also revealed that, apart from these studies, not much research has been conducted in other parts of the world and ascribed this to insufficient rainfall-monitoring networks and a lack of short duration (sub-daily) rainfall data. The inconsistency present in the ARF results obtained from these data-intensive empirical and/or analytical methods used internationally is also a major concern and could be ascribed to the variation in predominant weather types, storm durations, seasonal factors and recurrence interval (Skaugen, 1997; Asquith and Famiglietti, 2000; Allen and DeGaetano, 2005).

According to Svensson and Jones (2010), the level of agreement between the empirical and analytical methods currently in use is limited to a specific scaling regime, namely, short storm durations and small catchment areas. Thus, these methods are inappropriate for use with a comprehensive set of temporal and spatial scales such as at a QC level. On the other hand, a number of these empirical (storm-centred) and analytical (correlation-based and annual maxima-centred) methods do not provide probabilistically correct areal design rainfall estimates since it is assumed that the AEP of both the point and areal design rainfall is similar (Svensson and Jones, 2010).

Most of these methods are also based on a limited amount of observed rainfall data and are based on assumptions that are not entirely true to the actual rainfall process (Svensson and Jones, 2010).

The empirical ARFs used in South Africa are only based on the limited research conducted by Van Wyk (1965), Wiederhold (1969) and Alexander (1980). In the latter research attempt, the UK FSR (NERC, 1975) methodology was used as the basic approach, while the observed daily rainfall data was limited to the 1980s. There has also been a concern in some sections of the hydrological community in South Africa that the UK FSR results may not be appropriate for South African conditions (Van der Spuy and Rademeyer, 2014). Moreover, some studies (Omolayo, 1993; Siriwardena and Weinmann, 1996) have conclusively shown that ARFs are dependent on the average AEP of rainfall.

According to Svensson and Jones (2010), the use of radar rainfall information is currently problematic due to the heterogeneity and short recording period of the data, which might result in possible biases in ARF estimates. However, the use of weather radar systems in South Africa in recent years, together with improvements in rainfall analysis techniques, in *i.e.* merged observed rainfall data and radar images (Frezghi and Smithers, 2008) and the disaggregation of daily rainfall into hourly rainfall information (Knoesen and Smithers, 2008), could make the reassessment of ARFs using radar rainfall information a viable proposition in South Africa for the first time.

Based on the above-mentioned statements, it can be concluded that the development of ARFs appropriate to South Africa is a high-priority research area in design flood estimation. The ARFs in South Africa need to be reinvestigated in light of recent extreme flood events utilising longer periods of recording (45 years of additional data since the 1970s) which are now available for analysis. The variation of ARFs with recurrence interval, duration and rainfall-producing mechanisms also need to be investigated.

The overall purpose of the study, research aims, assumptions and specific objectives are discussed in the next section.

1.3 Purpose of Study

The overall purpose of this study is to develop an enhanced methodology to express the spatial and temporal rainfall variability at a QC level by means of probabilistically correct ARFs.

Firstly, an investigation was conducted into spatial and temporal rainfall variability in the 23 QCs present in the C5 secondary drainage region, which is selected as the study area. This investigation is conducted to derive probabilistically correct ARFs, these being ARF values expressed as a function of the catchment area (*A*), Mean Annual Precipitation (MAP), storm duration (*D*) and RI. Secondly, the derived ARFs are compared to a selection of ARF methods currently used in South Africa and internationally in order to assess the performance of the derived ARF values in 12 gauged catchments present in the C5 secondary drainage region.

1.3.1 Research aims

The primary aim of this study is to derive geographically-centred ARFs representative of the different rainfall-producing mechanisms at a QC level in the C5 secondary drainage region. The focus is on the development of probabilistically correct ARFs, namely, the relationships between *T*-year (RI) areal design rainfall estimates and weighted average *T*-year (RI) design point rainfall estimates for various *A*, MAP, *D* and RI values at a QC level are to be established.

1.3.2 Assumptions

This study is based on the following assumptions:

> **Assumption 1:** Design point rainfall estimates are only representative of a limited area while for larger areas the areal average design rainfall depth or intensity is likely to be less than the maximum design point rainfall depths or intensities.
>
> **Assumption 2:** ARFs vary with predominant weather types, storm durations, climatological factors and recurrence intervals.

Assumption 3: The current South African ARF estimation methods are only applicable to specific temporal and spatial scales.

1.3.3 Specific objectives

To achieve the research aims, the specific objectives are to:

(a) Analyse daily rainfall data for a range of storm durations (1 hour to 7 days), recurrence intervals (2 to 200 years) and catchment areas (100 to 10 000 km²).

(b) Evaluate and select one of the different averaging methods, *i.e.* conventional methods (such as the Arithmetic mean, Thiessen polygon and Isohyetal methods) or more sophisticated deterministic interpolation methods (such as the Inverse Distance Weighted (IDW) and Spline techniques) or geostatistical interpolation methods (such as the Kriging) to convert the point AMS rainfall to average point AMS rainfall values.

(c) Estimate improved ARFs using a modified version of Bell's method (1976).

(d) Establish mathematical relationships between the estimated ARFs and climatological variables (*e.g.* spatial and temporal rainfall distribution), catchment geomorphology (*e.g.* area, shape and geographical location) and/or a combination of these within the selected QCs. The results will then be presented in a suitable format which will be useful to practising engineers or hydrologists, *i.e.* a set of design curves and/or associated ARF algorithms.

(e) Compare the derived ARFs with a selection of empirical and/or analytical ARF methods currently used in South Africa and internationally in order to establish the relevance thereof and to assess the performance of the derived ARF values in gauged catchments.

1.4 Outline of Dissertation Structure

Each chapter is mostly self-contained and generally focuses on the use of ARFs to illustrate the temporal and spatial distribution of rainfall when the relationships between point and areal design rainfall are estimated.

Chapter 2 contains a literature review of the ARF estimation methods used internationally. The climate of South Africa, with associated rainfall types and an overview of the current status of observed rainfall measurement in South Africa are also included and discussed in Chapter 2.

An overview of the location and characteristics of the study area (C5 secondary drainage region) is provided in Chapter 3.

In Chapter 4, the detailed methodology adopted in meeting the specific objectives of this study is discussed. The focus in this chapter is on the extraction and analyses of observed daily rainfall data, averaging of observed point rainfall over an area, the probabilistic analysis for design rainfall estimation and the derivation of geographically-centred ARFs.

Chapter 5 presents the results based on the methodology followed in Chapter 4, with some further discussions included in Chapter 6.

The final conclusions and recommendations are also included in Chapter 6.

CHAPTER 2 : LITERATURE REVIEW

The literature review in this chapter focuses on the methods developed nationally and internationally to estimate ARFs. However, the climate of South Africa and associated rainfall types are discussed first. This is followed by an overview of the current status of observed rainfall measurement in South Africa. Thereafter, design rainfall estimation, some design point rainfall averaging techniques and a summary of the factors influencing ARFs are discussed.

2.1 Climate and Rainfall Types

The climate is highly variable in South Africa. Hence, hydrological and climatological information were used by Alexander (2010) to define nine distinctive climatological regions in South Africa as illustrated in Figure 2.1. Typically, apart from climate, other factors such as geographical location, altitude above mean sea level, rainfall type (convective, frontal and/or orographic), rainfall seasonality (summer, winter and/or all year) and average catchment slope classes (flat, moderate or steep) were also considered to define the various regions as shown in Figure 2.1. The study area is situated in the Highveld region.

Typically, in the south-western Cape (Mediterranean, and Southern Coastal regions), the climate is characterised by winter rainfall and warm windy summers, while highly variable, non-seasonal rainfall and extreme temperatures occur in the Karoo (KAR) region. Hot summers with convective thunderstorms and cold winters are typical on the Highveld, while mesic-subtropical conditions dominate on the KwaZulu-Natal coast of the Escarpment region (Davies and Day, 1998; Alexander, 2010). The MAP decreases, while potential evaporation increases westwards and northwards across South Africa. The overall MAP is 452 mm but in many parts of the country the MAP is much less. Evaporation exceeds rainfall throughout the country except in the mountainous Escarpment and Mediterranean regions. In the central parts of South Africa, evaporation is approximately twice the rainfall, while in the western parts of the country evaporation exceeds the rainfall by a factor of ten (Davies and Day, 1998).

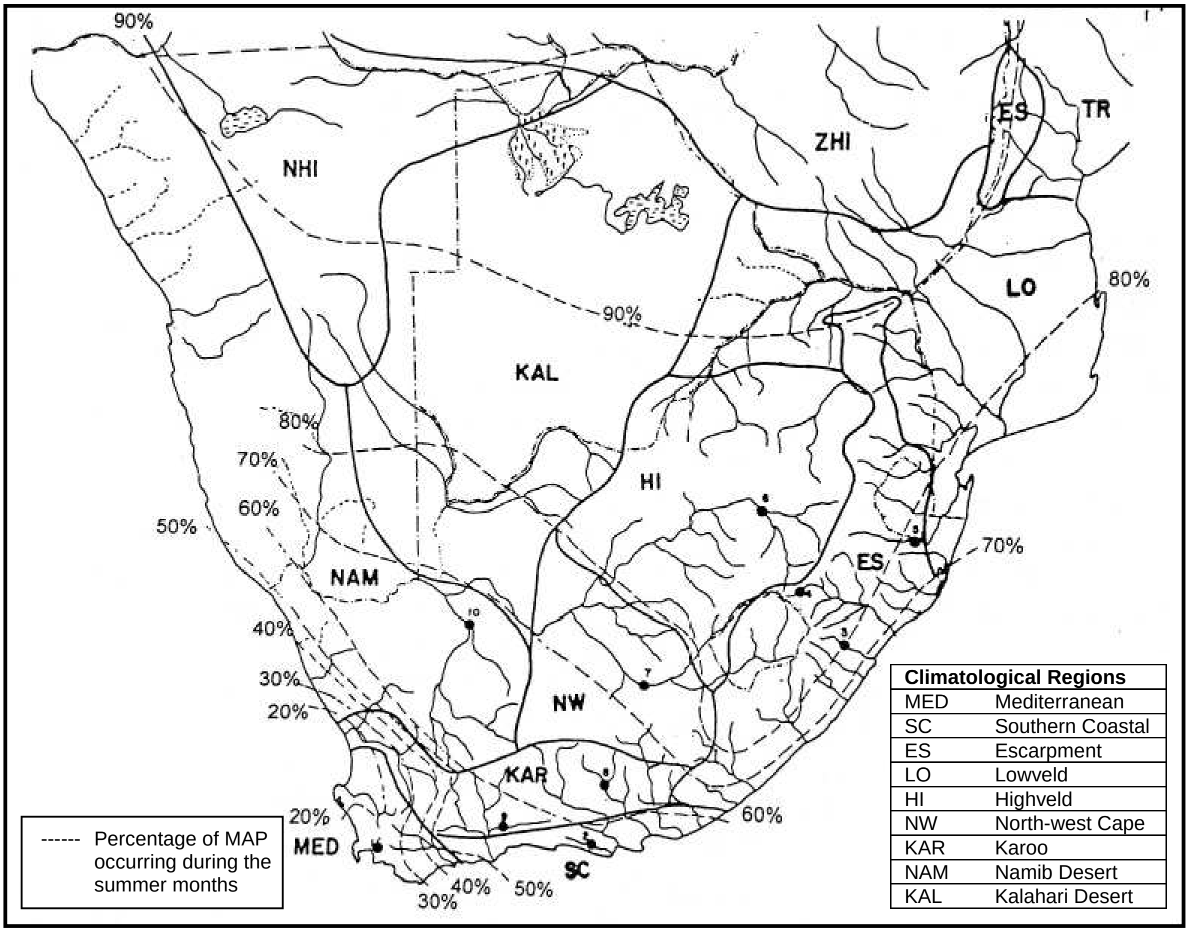

Figure 2.1: Climatological regions for South Africa (Alexander, 2010)

The temporal and spatial distribution of rainfall is highly variable on a seasonal and annual basis, since the rainfall is produced by different weather systems in different regions and at different times of the year (Davies and Day, 1998). In winter, the prevailing north-westerly winds result in high rainfall in the western part of the country, while the southern interior and Karoo remain dry. Summer rainfall is normally higher in the north and east, but due to dry high-pressure air masses that persist for long periods, the rainfall is low in the western parts of the country (Davies and Day, 1998).

Climate does not only affect rainfall distribution, but also rainfall intensity, duration and variability, which are all interdependent. However, the four major rainfall processes occurring in South Africa will also affect this interdependency, and are most likely to have different influences on the estimation of ARFs. The four major rainfall processes occurring in South Africa can be summarised as follows (Haarhoff and Cassa, 2009; Van der Spuy and Rademeyer, 2014):

(a) **Convective rainfall:** This process typically occurs during the summer season when air layers (closest to the earth's surface) saturated with water vapour are heated and subsequently tend to rise and cool down, resulting in cloud formation and rainfall. The rainfall intensity is normally high to very high with associated thunder activity. Convective rainfall is characteristic of the Highveld region which covers the Free State, Gauteng and Mpumalanga provinces.

(b) **Cyclonic rainfall:** This rare process typically occurs over the open sea and is formed when cyclones (large circular patterns) are growing in size, allowing moist air to be drawn into the cyclone vortex and allowing mist to be lifted up into the centre resulting in very strong winds and extremely high rainfall intensities.

(c) **Frontal rainfall:** This inland process typically occurs when cold or warm fronts are moving across the country and interact with one another. The cold air has the tendency to move underneath the warm air, and the warm air is deflected upwards by the trailing edge of the cold air. In both cases, the warm air is lifted up into the colder region, resulting in rainfall.

(d) **Orographic rainfall:** This process usually occurs near coast lines and typically develops when wind blows over the open sea towards land carrying air saturated with water vapour until it reaches a mountain range. At these geographical barriers, the saturated air is forced upwards to result in condensation and rainfall. The rainfall intensity is normally regarded as moderate and dependent on wind blowing towards the inland areas. Orographic rainfall is characteristic of the coast lines of KwaZulu-Natal, and the Western Cape Province.

The rainfall types (a) to (d) were considered carefully to highlight and describe the direct influence of these on the estimation of ARFs. The magnitude of ARFs is highly dependent on the different storm mechanisms associated with different rainfall types. In a specific region with more frequent thunderstorms (convective rainfall) occurring than frontal storms (wide spread rainfall), the typical observed point rainfall Annual Maximum Series (AMS) for that specific region would likely consist of rainfall values associated with convective activity (rainfall with rapidly changing intensity), whereas the frontal rainfall values could have been more representative of the actual rainfall process in that particular catchment or region. This may result in much lower probabilistically correct ARFs (thunderstorms with high intensities), opposed to the probabilistically higher ARFs represented by the frontal activity (Siriwardena and Weinmann, 1996).

In recognition of the above-mentioned interdependencies, Weddepohl (1988, cited by Schulze *et al.*, 1992) demarcated South Africa into four distinctive daily rainfall intensity distribution regions. Typically, Region 1 is associated with a Type 1 design rainfall intensity distribution which is regarded as the lowest, while Type 4 is associated with the highest rainfall intensity. The spatial distribution of these regions can be summarised as follows: (i) Region 1: Eastern Cape, namely, East London and Port Elizabeth; (ii) Region 2: Western Cape (Karoo) and Free State; (iii) Region 3: Northern Cape, namely, Upington and Kimberley, as well the Highveld, namely, Gauteng and Mpumalanga; and (iv) Region 4: the remainder of the country (Weddepohl, 1988, cited by Schulze *et al.*, 1992).

2.2 Observed Rainfall Data in South Africa

Observed rainfall data in South Africa can be obtained from daily rainfall stations, which are widespread in space, and are measured as a depth (mm) at a specific time interval on each day. The poor maintenance of rainfall stations in South Africa, under the supervision of the South African Weather Services (SAWS), was highlighted by Smithers and Schulze (2000b) and confirmed in a more recent study by Van Vuuren *et al.* (2012). The recent survey highlighted that approximately 1 200 rainfall stations are currently out of service whereas most of these stations were operational in the late 1960s. Unfortunately, the current number of operational rainfall stations is less than in the 1920s and, considering this trend, South Africa might have even fewer operational rainfall stations in the near future (Smithers and Schulze, 2000b; Van Vuuren *et al*., 2012).

A total of 11 171 daily rainfall record lengths (long duration) are available in the South African database (Smithers and Schulze, 2000b). This is illustrated in Figure 2.2. The SAWS contributed 78.9% of the data, followed by the Institute for Soil, Climate and Water (ISCW) (7.7%), joint SAWS-ISCW (3.3%), South African Sugar Association Experiment Station (SASEX) (1.4%) and the remaining 8.8% by private entities (Smithers and Schulze, 2000b). However, more than 20% of all daily rainfall stations with record lengths exceeding 20 years have more than 10% of their data missing (Smithers and Schulze, 2000b).

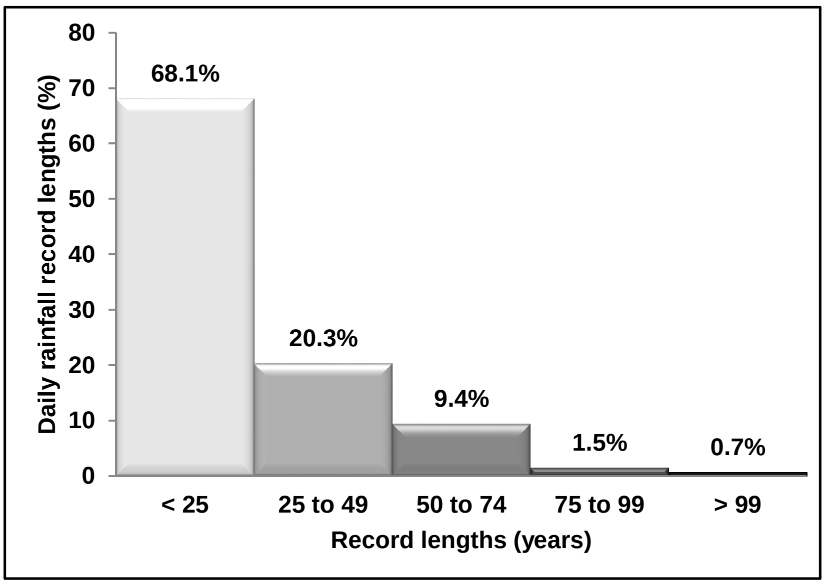

Figure 2.2: Available record lengths for daily rainfall stations in South Africa (after Smithers and Schulze, 2000b)

Short duration rainfall data (less than 24 hours) in South Africa are currently available from 412 stations as shown in Figure 2.3. However, only 49 of these 412 rainfall stations have record lengths exceeding 30 years or longer (Smithers and Schulze, 2000a). The SAWS was the largest contributor to this sub-daily rainfall database, *i.e.* 81% of all stations (Smithers and Schulze, 2000a).

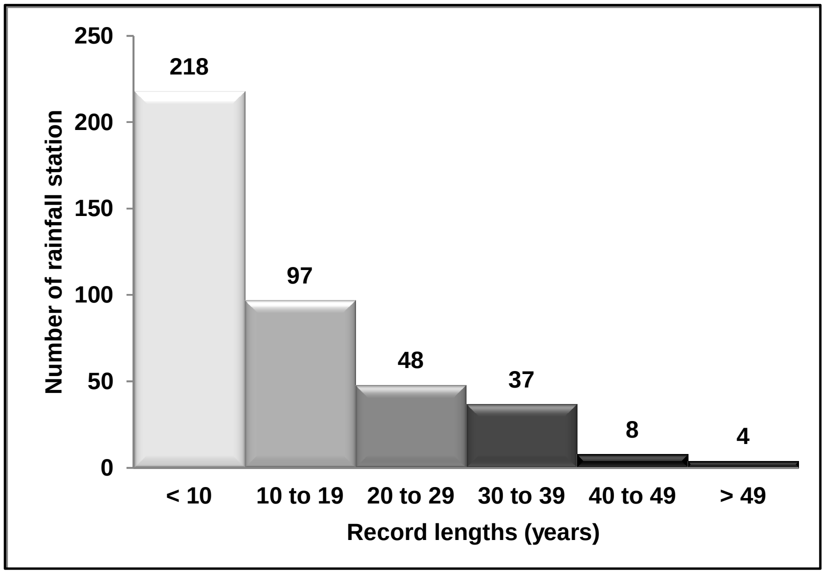

Figure 2.3: Short duration rainfall stations in South Africa (Smithers and Schulze, 2000a)

Furthermore, Smithers and Schulze (2000a) also highlighted that short duration rainfall data have a low reliability due to several possible errors including missing data and differences (more than 20 mm) between the digitised and standard rain gauge daily totals. It was also noted that the digitised SAWS data is inadequate for the estimation of design storm durations of less than 24 hours. Smithers and Schulze (2000a) developed three approaches based on regional similarities, scaling properties and stochastic simulation of extreme rainfall events to estimate short duration design rainfall values. This is discussed further in the following section.

2.3 Infilling of Missing Observed Rainfall Data

Rainfall records characterised by missing data are a serious concern when daily hydrometeorological simulation models are used since all these models are reliant on a continuous rainfall data series input (Pitman, 2011).

Lynch (2004) highlights the importance of rainfall data infilling and emphasises that a missing day implies an incomplete month and consequently an incomplete year. Lynch (2004) proposes a number of different infilling techniques based on a categorisation process and developed a Daily Rainfall Extraction Utility (DREU) to determine the best approach to infill any missing data at rainfall station(s). The DREU infilling procedure algorithms are based on one or a combination of the following techniques:

(a) **Inverse Distance Weighting (IDW):** The IDW technique (Meier, 1997; cited by Lynch, 2004) inversely weights the rainfall records from rainfall stations surrounding the rainfall station under consideration, depending on the distance of those rainfall stations from the rainfall station under consideration. Meier (1997; cited by Lynch, 2004) established a procedure for selecting neighbouring rainfall stations from each quadrant around the rainfall station under consideration. This approach ensured that a certain number of rainfall stations are selected from each of the four quadrants surrounding the station in order to minimise the uncertainty introduced when the closest few rainfall stations are all in the same direction from the rainfall station under consideration (Meier, 1997; cited by Lynch, 2004).

(b) **Expectation Maximisation Algorithm (EMA):** The EMA technique was adopted and refined by Makhuvha (1997a, 1997b, cited by Lynch, 2004) to infill missing rainfall data on a monthly basis. The EMA technique revolves around a recursive action of substituting missing data in a multiple linear regression relationship to re-estimate the values between the data at the rainfall station under consideration and the data from the nearby control rainfall stations. Smithers and Schulze (2000b) highlight that the EMA technique requires the selection of suitable control rainfall stations be valuable in determining the suitability of using the selected target and control rainfall stations for the simultaneous infilling of missing data.

(c) **Median Ratio (MR) technique:** The MR technique depends on the median values between the rainfall station under consideration and the nearest control rainfall station to estimate a proportionality ratio. The latter proportionality ratio is used to correct the data from the rainfall station under consideration and to infill the missing daily data series. The advantage of the MR technique is that the closest control rainfall station with non-existing data will be replaced by the second closest control rainfall station (Lynch, 2004).

(d) **Monthly Infilling (MI) technique:** A regression approach was used to fill in the non-existing missing monthly rainfall data by using the surrounding control rainfall stations as described by Zucchini (1984, cited by Lynch, 2004). The monthly database (observed and infilled) by Dent (1989, cited by Lynch, 2004) was interrogated and the monthly infilled values of zero and/or ≤ 2 mm were extracted.

The EMA and MR techniques are considered to be the most effective infilling techniques in the DREU (Lynch, 2004). Any missing observed rainfall values not infilled by using the EMA and MR techniques are infilled using the IDW technique. Consequently, zero and less than 2 mm rainfall values, as derived by Dent (1989, cited by Lynch, 2004), are then used to infill any remaining missing values that have not been infilled. The South African daily rainfall database has more than doubled in size with the infilling techniques described above. The rainfall database consists of 105 753 218 daily observed values with 236 154 934 infilled values (Lynch, 2004). The observed and infilled rainfall database therefore has 341 908 152 values (Lynch, 2004).

2.4 Averaging of Observed Rainfall

In the assessment of total quantities of rainfall over large areas, the occurrence of storms and their contribution to single rainfall stations is unknown. Therefore, it is necessary to convert numerous observed point rainfall depths to provide an average rainfall depth over a certain area. The following methods may be used for averaging the rainfall depth over an area (Wilson, 1990):

(a) **Arithmetic mean method:** This method [Eq. (2.1)] is defined as the sum of all the point rainfall information divided by the number of rainfall stations within the catchment area. This method is only sufficient when rainfall stations are uniformly distributed, the topography is relatively flat and spatial variations in rainfall are insignificant.

$$\overline{P} = \sum \frac{P_i}{N_i} \qquad (2.1)$$

Where:

$\overline{P}$ = spatial average rainfall depth (mm),

N_i = number of rainfall stations within catchment area, and

P_i = point rainfall depth (mm).

(b) **Thiessen polygon method:** This method [Eq. (2.2)] defines the zone of influence of each rainfall station by drawing lines between pairs of stations, bisecting the lines with perpendiculars. The total area enclosed within the boundary formed by these intersecting perpendiculars has rainfall of the same amount as the enclosed rainfall station. This method is not suitable for mountainous areas due to the orographical influences.

$$\overline{P} = \sum \frac{A_s P_i}{A_T} \tag{2.2}$$

Where:

$\overline{P}$ = spatial average rainfall depth (mm),

A_s = area of the sub-catchment contributing to the rainfall station (km²),

A_T = total catchment area (km²), and

P_i = point rainfall depth (mm).

(c) **Isohyetal method:** This method [Eq. (2.3)] is based on the interpolation between rainfall stations to produce isohyets or contours of equal rainfall depth. The areal average of the weighted rainfall depths between the isohyets is then used to determine the average rainfall. This method is possibly the most accurate with an added advantage that the isohyets may be drawn to take into account local effects of climate and uneven topography.

$$\overline{P} = \frac{\sum P_i A_i}{\sum N_i} \tag{2.3}$$

Where:

$\overline{P}$ = spatial average rainfall depth (mm),

A_i = area (km²),

N_i = number of rainfall stations within area, and

P_i = point rainfall depth (mm).

(d) **Grid point method:** For the grid point method, a uniform grid is superimposed over a catchment area containing the spatial location of each rainfall station (and associated rainfall depths). Rainfall is estimated at each corner of the grid and then multiplied with the representative grid-area to obtain the average rainfall volume. The sum of all the estimated volumes divided by the total catchment area equals the average areal rainfall depth (Patra, 2008).

(e) **Isopercental method:** The Isopercental method is very similar to the Isohyetal method but is preferable when dealing with orographic and other topographic differences in mountainous areas. For this method, a catchment map containing the spatial location of each rainfall station and associated rainfall depths (daily, monthly and annually) must be available. The rainfall data (daily or monthly) is expressed as a percentage of the annual rainfall values to produce isopercental lines. The isopercental lines with the same percentage value and intervals should join each other over the catchment. Isohyetal lines, representative of the annual rainfall values in the region, must be drawn to overlay the isopercental lines. The intersecting points between the isopercental and isohyetal lines are then used to estimate the rainfall. Consequently, the areal rainfall over the catchment could be estimated in a similar fashion to the Isohyetal method. However, it should be noted that this method is difficult to implement and is regarded as data intensive (Patra, 2008).

(f) **IDW method:** This method [Eq. (2.4a)] is based on deterministic interpolation and takes the geographical position of each rainfall station relative to the other rainfall stations into consideration. A rainfall station which is geographically distant/close to other stations will have a larger/smaller weighting factor [Eq. (2.4b)] and will therefore contribute more/less to the estimation of the average areal rainfall. In essence, the sum of all point rainfall information is multiplied with individual weighting factors and divided by the total number of rainfall stations within the catchment under consideration (ESRI, 2006; Dyson, 2009).

$$\overline{P} = \frac{\sum P_i W_i}{N} \tag{2.4a}$$

$$W_i = \frac{\sum_{m=1}^{N-1} r_{\max}}{(N-1) r_{\max}} \tag{2.4b}$$

Where:

$\overline{P}$ = spatial average rainfall depth (mm),

W_i = individual weighting factor,

m = rank value of individual weighting factors,

N = total number of rainfall stations,

P_i = point rainfall depth (mm), and

r_{max} = maximum distance between the specific rainfall station and any other rainfall station (m or km).

(g) **Spline method:** The Spline method is also based on deterministic interpolation, which provides a smooth rainfall surface based on the point rainfall values as primary input. In other words, it fits a mathematical function to a specified number of nearest input points while passing through the sample points. This method is recommended for generating gently varying rainfall surfaces, such as frontal rainfall distributed over larger areas as opposed to highly variable, localised convective rainfall (ESRI, 2006).

(h) **Kriging method:** Kriging is based on a geostatistical interpolation process utilising auto-correlation, such as the statistical relationships among point rainfall values. Kriging does not only have the capability of producing a rainfall prediction surface, but also provides some measure of the certainty of the predictions. The variation in the rainfall surface can be explained by the distance or direction between the rainfall stations that reflect correlation. The average rainfall for each location is determined by a mathematical function applied to the number of rainfall stations within a catchment or specified radius. The use of Kriging is recommended when the rainfall information is characterised by a spatially correlated distance or directional bias (ESRI, 2006).

The estimation of design rainfall is discussed in the next section.

2.5 Design Rainfall Estimation

Design rainfall comprises of a depth and duration associated with a given recurrence interval or AEP (Smithers and Schulze, 2004). Short and long duration design rainfall estimations can either be based on point or regionalised data. Rainfall durations less than 24 hours are generally classified as short, while long durations typically range from 1 to 7 days (Smithers and Schulze, 2004).

Several regional and national scale studies in South Africa based on short durations and point data were conducted between 1945 and 2001. Studies focusing on long durations based on daily point rainfall data included studies done by the SAWB (South African Weather Bureau), Schulze (1980), Adamson (1981), Pegram and Adamson (1988) and Smithers and Schulze (2000b). Smithers and Schulze (2000a; 2000b) also used a regionalised approach in an attempt to increase the reliability of the design values at gauged sites, as well as for the estimation of design values at ungauged sites (Smithers and Schulze, 2003).

2.5.1 Single site approach

A single site approach requires that each rainfall station within the relevant catchment be investigated to determine the record length, data quality (errors, missing data and outliers) and topographical position (Smithers and Schulze, 2000a). In order to develop the depth-duration-frequency (DDF) relationship at every single site, the following steps are of importance (Smithers and Schulze, 2000a):

(a) Selection of the most appropriate data set. This may either be the AMS or Partial Duration Series (PDS) with a sufficient record length;

(b) Selection of the most appropriate probability distribution; and

(c) Selection of a suitable parameter and quantile method.

These steps are discussed in more detail in the following paragraphs.

A probabilistic analysis needs to be conducted at each rainfall station and it is thus advisable not to use rainfall stations with short record lengths. Furthermore, it is impossible to conclusively select a distribution that could consistently provide adequate rainfall frequency estimates for recurrence intervals greater than the period of record. On the other hand, small samples may define a distribution which is markedly different from the parent population (Smithers and Schulze, 2000a).

According to Viessman *et al*. (1989), a minimum record length of 10 years is required, while Schulze (1984) questioned the significance of the record length for extreme events recorded and hence the design values. Hogg (1992) demonstrated that even 20 years of data is not stable enough to estimate the 10-year return period event. Hogg (1992) indicated that the assumptions of stationarity and homogeneity of the AMS of rainfall are seldom valid. It is suggested that a regional approach be used to improve the frequency analysis of extreme rainfall events.

According to Weddepohl (1988), the malfunctioning of rainfall gauges and processing errors are inherent in rainfall data. The spatial density and distribution of rainfall gauges, sporadic rainfall events as opposed to the continuous digitised data in use, the length of available records and the presence of outliers are all problems associated with these errors (Weddepohl, 1988).

The selection of the most suitable probability distribution resembling the probability distribution of the population must be made according to the theoretical basis, consistency, acceptance, user-friendliness and applicability thereof (Cunnane, 1989, cited by Smithers and Schulze, 2000a). This selection is particularly important when estimating extreme events with recurrence intervals greater than the length of record. Equally important is that factors such as the type of data in use, data stationarity and the method of fitting the distribution also be considered (Cunnane, 1989, cited by Smithers and Schulze, 2000a).

The Extreme Value Type I (EV1) distribution has been extensively used in rainfall DDF studies in South Africa since 1963, while the use of the integrated General Extreme Value (GEV) distribution is growing in the application of frequency analysis. Van der Spuy and Rademeyer (2014) propose the use of the Log-Normal (LN), Log-Pearson Type III (LP3) as well as GEV using the Method of Moments (MM), Probability Weighted Moments (PWM) or Linear Moments (LM) to estimate the required design rainfall depths in South Africa.

2.5.2 Regional approach

Regional frequency analysis is based on the assumption that the standardised variate distributions of rainfall data are similar at every single site in a region and that the data from various single sites in a region can thus be combined to generate a single regional rainfall frequency curve representative of any site in the specific region with appropriate site-specific scaling. An advantage of this approach is that it can be used to estimate events at ungauged sites where no rainfall data exists (Alexander, 2001; Cunnane, 1989, cited by Smithers and Schulze, 2003).

In nearly all practical situations, a regional approach is preferred to a single site approach primarily based on the efficiency and accuracy of the rainfall quantile estimation and where statistical homogeneity or heterogeneity might exist (Hosking and Wallis, 1997, cited by Smithers and Schulze, 2003). The large degree of uncertainty introduced in the extrapolation of AEPs beyond the record length of data can also be reduced by regionalisation since the observed rainfall at a single site is then related to the hydrological response at a regional scale by making use of an extended or combined record length of data (Smithers and Schulze, 2003).

Regional approaches are well established in frequency analysis and various different techniques are available. The two design rainfall databases discussed below are generally used by practising engineers and hydrologists in South Africa. The second database (b) makes use of a regional index-rainfall type approach.

(a) **Technical Report (TR) 102:** The 1, 2, 3 and 7-day extreme design rainfall depths for recurrence intervals of 2, 5, 10, 20, 50, 100 and 200 years were estimated by Adamson (1981) using approximately 1 946 rainfall stations. A censored LN distribution based on the PDS was used to estimate the design rainfall depths at a single site.

(b) **Regional Linear Moment Algorithm (RLMA)-SAWS:** Smithers and Schulze (2000b) conducted frequency analyses based on the GEV probability distribution at 1 789 rainfall stations with at least 40 years of records to estimate the 1-day design rainfall values in South Africa. This was followed by a regionalisation process (based on LM estimators) and the establishment of 78 relatively homogeneous rainfall regions and associated index values derived from at-site data (Smithers and Schulze, 2000b). Quantile growth curves, representative of the ratio between design rainfall depth and an index storm to recurrence interval, were developed for each of the homogeneous rainfall regions and storm durations of 1 to 7 days. These regionalised growth curves and at-site index values were then used to estimate design rainfall depths at 3 946 rainfall stations in South Africa. The RLMA&SI is currently the recommended method of estimating design rainfall in South Africa (Smithers and Schulze, 2000b).

The sampling variability of the AMS was estimated using three approaches: (i) windows of data extracted from the entire period of record, (ii) stochastic modelling and (iii) a bootstrapping technique (Smithers and Schulze, 2003; 2004). The results established that the variation with duration in observed higher order LM is associated with the sampling variability and record length. Based on the fact that the daily rainfall data is more reliable with longer record lengths than the digitised data, the 1-day LM ratios were assumed to be the most reliable estimate of the LM ratios and mean AMS for all durations (Smithers and Schulze, 2003; 2004).

The mean AMS for any duration can be estimated by estimating the mean 1-day AMS at a single site by regional regression and then scaling either the mean AMS for durations shorter or longer than one day respectively from the 24-hour and 1-day values (Smithers and Schulze, 2003). This application of the RLMA in conjunction with a scale invariance (SI) approach is referred to as the RLMA&SI approach (Smithers and Schulze, 2003).

The RLMA&SI approach was compared with the TR102 single site approach at 2 184 daily rainfall stations (Adamson, 1981). The differences between the two approaches were generally less than 20% for recurrence intervals less than 50 years but these differences increased for larger recurrence intervals. These differences can be ascribed to the following factors (Smithers and Schulze, 2003):

(a) The use of longer record lengths and strict data quality control procedures in the RLMA&SI approach;
(b) The use of different probability distributions – LN in TR102 as opposed to the GEV in RLMA&SI; and
(c) The LM values used in the RLMA&SI approach are not that sensitive or influenced by outliers in data.

Further comparisons were performed between the: (i) RLMA&SI approach, (ii) DDF relationships based on Log-Extreme Value Type I (LEV1) distributions fitted to the AMS as contained in the Hydrological Research Unit (HRU) Report 2/78 (Midgley and Pitman, 1978), (iii) Hershfield equation (Adamson, 1981), and (iv) modified Hershfield equation (Alexander, 2001) for durations less than 24 hours. The design rainfall estimation results obtained with RLMA&SI and DDF approaches compared well, while the modified Hershfield equation generally overestimated the values and there were also inconsistencies between the modified Hershfield equation and the 1-day TR102 information. The functional relationship of the modified Hershfield equation does not seem to accommodate the curvilinear relationship between the design rainfall depth and $\log_{10}$-transformed duration (Smithers and Schulze, 2003).

In conclusion, the RLMA&SI estimates proved to be consistent over the entire range of durations, whereas the other techniques considered are frequently inconsistent for a range of durations (Smithers and Schulze, 2003; 2004). The software program, *Design Rainfall Estimation in South Africa*, was developed in 2003 to facilitate the estimation of design rainfall depths at a spatial resolution of 1-arc minute for any location in South Africa based on the RLMA&SI approach for durations ranging from 5 minutes to 7 days and for recurrence intervals of two to 200 years (Smithers and Schulze, 2003; 2004).

Irrespective of whether a single site or regional approach is adopted, the design rainfall depth to be used in design flood estimation, especially in the deterministic methods, must be based on the critical storm duration or time of concentration (T_C) of a catchment. Thus, depending on the T_C, the daily design rainfall depth used in flood estimations must either be increased or decreased. In order to convert the daily design rainfall depth values to independent durations of the same length, conversion factors have to be used. The conversion factors are dependent on the duration in question and various values have been proposed. The factors recommended to convert daily design rainfall depths to 24-hour continuous maxima are 1.13 in the USA (Hershfield, 1962), 1.06 in the UK (NERC, 1975), 1.13 (Alexander, 1978) and 1.11 (Adamson, 1981) in South Africa (Smithers and Schulze, 2000a; Alexander, 2001).

However, the latter South African approaches are regarded as outdated and Smithers and Schulze (2000a) developed regionalised relationships for 15 relatively homogeneous rainfall regions in South Africa, with a national average of 1.21.

The above-mentioned conversion factors applicable to the summer/inland and winter/coastal rainfall regions of South Africa are listed in Table 2.1 (≤ 24 hours) and Table 2.2 (daily to hourly).

Table 2.1: Ratio of *Tc* (hours) storm depth to 24 hour storm depth (Adamson, 1981)

T_C (hours)	Summer/inland region	Winter/coastal region
0.10	0.17	0.14
0.25	0.32	0.23
0.50	0.46	0.32
1	0.60	0.41
2	0.72	0.53
3	0.78	0.60
4	0.82	0.67
5	0.84	0.71
6	0.87	0.75
8	0.90	0.81
10	0.92	0.85
12	0.94	0.89
18	0.98	0.96
24	1.00	1.00

Table 2.2: Conversion of fixed time interval rainfall measurement to continuous rainfall measurement (Van der Spuy and Rademeyer, 2014)

Duration		Conversion factor
From (days)	To (hours)	
1	24	1.11
2	48	1.07
3	72	1.05

Table 2.2: (continued)

Duration		Conversion factor
From (days)	To (hours)	
4	96	1.04
5	120	1.03
7	168	1.02
> 7	> 168	1

2.5.3 Other approaches

Apart from the approaches discussed in Sections 2.5.1 and 2.5.2, four different approaches are also used by the Department of Water and Sanitation (DWS) to estimate catchment design rainfall (Van der Spuy and Rademeyer, 2014). The first approach, referred to as the 'Smithers Regional Rainfall (SRR)' approach, is in essence the RLMA&SI approach as discussed in Section 2.5.2. The remaining three approaches are summarised as follows (Van der Spuy and Rademeyer, 2014):

(a) **Maximum Station Rainfall (MSR_{SS}) approach:** The rainfall data at a single rainfall station is probabilistically analysed by using either the observed or infilled rainfall data series. This approach is similar to a conventional single site approach as discussed in Section 2.5.1.

(b) **Maximum Station Rainfall (MSR_{CS}) approach:** The weighted AMS catchment rainfall data based on either the full observed or infilled record length (if applicable) are probabilistically analysed. The use of an infilled record length ensures that the longest possible record length is utilised in the analysis.

(c) **Daily Catchment Rainfall (DCR) approach:** This approach requires the weighted point rainfall at a daily time interval within a specific catchment (Van der Spuy and Rademeyer, 2014). The weighted daily catchment rainfall is then probabilistically analysed to obtain areal design rainfall which incorporates the temporal and spatial variation of predominant weather types in a catchment. Van der Spuy and Rademeyer (2014) also highlighted that ARFs are not applicable to this approach since the areal design rainfall is already representative of a geographically-fixed area.

The factors which have an influence on the estimation of ARFs are discussed in the next section.

2.6 Factors Influencing ARFs

Numerous factors can have a significant influence on the estimation of ARFs such as climatological variables, catchment geomorphology, methodological approaches to estimate ARFs and/or a combination of these (Asquith and Famiglietti, 2000; Svensson and Jones, 2010). All these factors are discussed in this section to highlight their individual influences.

2.6.1 Climatological variables

Geographical location within different climatological regions has a direct influence on ARFs. It was established that the 1-day ARFs in the USA exceeded the equivalent ARF estimates in Australia, while the ARFs decline more rapidly in the semi-arid south-western USA than in the rest of the USA (Svensson and Jones, 2010). Similar trends were also confirmed by Asquith and Famiglietti (2000) who established that the ARFs are higher in the eastern USA than in Texas. ARFs are also influenced by seasonal variability, for example, higher values are obtained in winter than in summer. This could be ascribed to the response to increased convective activity in summer (Allen and DeGaetano, 2005).

Different rainfall-producing mechanisms, such as convective vs. frontal rainfall, will produce different spatial rainfall patterns. Typically, the spatial averages for large-scale frontal rainfall do not reduce much in magnitude with increasing area, whereas this is the case for small-scale convective rainfall events (Skaugen, 1997). Skaugen (1997) also established that ARFs for both convective and frontal rainfall decrease with an increasing recurrence interval, but the rate of decrease for convective rainfall is noticeably larger than that for frontal rainfall. The decrease in ARFs with increasing recurrence intervals may also reflect the importance of convection in producing very high point rainfalls. Huff and Shipp (1969) highlighted that the spatial correlation decay pattern of low-pressure centred storms is smaller compared to fronts associated with mid-latitude cyclones, while it is the greatest in air mass storms.

In the USA, areal rainfall was found to decrease with the corresponding point rainfall and with increasing recurrence intervals (Asquith and Famiglietti, 2000;

Allen and DeGaetano, 2005). In contrast, Grebner and Roesch (1997) demonstrated that ARFs in Switzerland (A > 4 500 km²) are independent of the recurrence interval. The ARFs contained in the UK FSR (NERC, 1975) decrease more rapidly (with increasing catchment areas) for shorter storm durations than for longer storm durations. It was also confirmed that the ARFs derived using a storm-centred approach are independent of the recurrence interval and geographical location (Svensson and Jones, 2010). Alexander (2001) recommends a geographically-centred approach when assuming a uniform spatial and temporal rainfall distribution for the total storm duration over the whole catchment area. Alexander (2001) also emphasises that practising engineers or hydrologists using storm-centred data to derive ARFs should not assume uniform rainfall intensity distribution over the catchment.

2.6.2 Catchment geomorphology

Most research conducted on the estimation of ARFs concluded that catchment geomorphology (such as area, shape and topography) has an insignificant influence on ARFs (Svensson and Jones, 2010).

In catchments with areas less than 800 km², ARFs are mainly a function of the area and point intensity since the relationship between rainfall intensity and the infiltration rate of the soil is predominant. In catchments with areas of up to 30 000 km², ARFs are mainly a function of the area and storm duration (Alexander, 2001; SANRAL, 2013). Lambourne and Stephenson (1986) demonstrated that the ARF will decrease from unity with an increasing catchment area. ARFs could also vary between urban areas and the surrounding rural areas. Huff (1995) showed that eight storms in Chicago, USA, had a slower ARF decreasing rate within 500 km² from the urban storm centre compared to 67 rural storms. Veneziano and Langousis (2005) concluded that the catchment shape normally has an insignificant effect on ARFs. However, different ARF estimates could be expected in catchments with an elongated shape where the rainfall distribution patterns and direction of movement are aligned along the catchment or perpendicular to it.

Topography (such as hills and mountains) has leeward and windward effects on rainfall and may affect ARFs. Rainfall-monitoring networks also tend to be sparser at higher altitudes; consequently resulting in poorer areal rainfall estimates. Nevertheless, Allen and DeGaetano (2005) found that topographical rainfall biases appear to be insignificant for the estimation of ARFs.

2.6.3 Methodological approaches

The record length of rainfall data and frequency of data collection may influence ARFs due to temporal rainfall variability. Asquith and Famiglietti (2000) showed that three overlapping rainfall-monitoring networks around Houston in Texas, USA did not yield the same ARFs due to different rainfall-monitoring networks that cannot be indiscriminately combined. However, Allen and DeGaetano (2005) showed that the density of rainfall-monitoring networks and the use of different interpolation methods have an insignificant influence on the estimation of ARFs in North Carolina and New Jersey, USA. Asquith and Famiglietti (2000) estimated probabilistically correct ARFs and proved that the recurrence interval has a significant influence on the relationship between the ratio of the annual maxima to concurrent rainfall depth and on the separation distance from the annual maxima point rainfall.

Unfortunately, the two recognised approaches, namely, the storm-centred and geographically-centred approaches, used to estimate ARFs generally provide inconsistent results. In using a storm-centred approach, the isohyets of a complete storm are analysed without considering the geographical location thereof (Alexander, 2001). In the case of a geographically-centred approach, storms occurring over a fixed area or collection of rainfall stations on the catchment's surface are considered (Alexander, 2001). Bell (1976) highlighted that the theoretical significance of the geographically-centred approach is more statistical than physical and is therefore best interpreted in terms of areal average point rainfall frequency curves, which simply provides the ratios of areal to point rainfall with the same AEP. It is thus quite evident that the use of different methodologies to estimate ARFs is likely to result in different ARF estimates.

The next section provides some background on the different categories of ARF estimation and contains a review of the methods used to estimate ARFs in South Africa. A detailed description of Bell's (1976) method follows since the ARF values derived in this study are based on a modified version of this method. The other ARF estimation methods used internationally are summarised in a tabular format and included as part of Tables A1 and A2 in Appendix A.

2.7 ARF Estimation Methods

The estimation methods for ARFs can be grouped into two broad categories as discussed below:

(a) **Empirical methods:** These methods can either be based on a geographically-centred or storm-centred approach. The geographically-centred approach describes the relationship between areal average design rainfall over a geographically fixed area (such as a catchment) and a corresponding design point rainfall value representative of the area under consideration. In other words, the ARF is used for percentage reduction, which relates to the statistics of point and areal design rainfall and considers the uniform temporal and spatial distribution of rainfall over the catchment area.

In the storm-centred approach, the region over which the areal design rainfall is estimated is not fixed but changes for each storm (Alexander, 2001; Svensson and Jones, 2010). The centre point for the approach is characterised by the maximum rainfall, which also changes for each storm. In other words, the ARF relates to the way in which rainfall intensity decreases with distance from the central core of individual storm events, with the average areal design rainfall intensity being estimated (Alexander, 2001; Svensson and Jones, 2010).

Sivapalan and Blöschl (1998) noted that storm-centred ARFs are usually somewhat smaller than geographically-centred ARFs as the ARFs are either derived from convective storms with heavy design point rainfall and a limited areal extent or the highest design point rainfall might be located outside the catchment boundary. Yoo, *et al.* (2007) describe a variant of the geographically-centred approach and suggest a mixed distribution based on the concept of rainfall intermittency (wet and dry periods, with a continuous gamma distribution fitted to the wet periods) for estimating rainfall recurrence intervals.

Methods such as the USWB method (USWB, 1957; 1958), UK FSR method (NERC, 1975) and Bell's (1976) method are typical examples of empirical methods. The latter method has proved to offer more probabilistically correct ARFs compared to the other methods since the AEP was incorporated. Each empirical method has different data requirements and subsequently the ARF estimates will also differ.

All these methods depend on the data availability, climatology, catchment geomorphology and rainfall characteristics (Svensson and Jones, 2010).

(b) **Analytical methods:** Derived mathematical algorithms are used to characterise the spatial and temporal rainfall variability by incorporating simplified assumptions that are not entirely true descriptions of the actual rainfall process (Siriwardena and Weinmann, 1996; Svensson and Jones, 2010). The fact that the actual rainfall processes are partially ignored is a cause for concern and this is further compounded by the often limited amount of actual rainfall data used during the verification of these methods.

However, with further verification, some of these methods might provide adequate ARF estimates. In response to these inherent shortcomings, several new analytical methods to estimate ARFs have been proposed during the last three decades such as storm movement (Bengtsson and Niemczynowicz, 1986), crossing properties (Bacchi and Ranzi, 1996), spatial correlation structure (Sivapalan and Blöschl, 1998) and scaling relationships (De Michéle *et al*., 2001).

2.7.1 South African methods

The methods used in South Africa to estimate ARFs are limited to the studies conducted by Van Wyk (1965), Wiederhold (1969) and Alexander (1980; 2001).

The details of each method are summarised as follows:

(a) **Van Wyk's method (1965):** The first South African attempt to analyse ARFs based on a storm-centred approach was conducted by Van Wyk (1965, cited by Lambourne and Stephenson, 1986) on a small-scale (catchment areas ≤ 800 km²) in Pretoria, Gauteng. In addition, a few rainfall storm areas from the USA and Canada were analysed for comparison purposes. Isohyetal maps of several storms were plotted based on the average areal rainfall depths in catchments ranging from 10 km² to 800 km² centred on the maximum point rainfall and expressed as a percentage of point rainfall at the storm centre (Van Wyk, 1965, cited by Lambourne and Stephenson, 1986). The ARFs were also expressed as a function of the point source rainfall intensity, particularly an average intensity over the storm duration at the storm centre (Van Wyk, 1965, cited by Lambourne and Stephenson, 1986). As a result, depth-intensity-area envelope diagrams were developed (Figure 2.4). This figure is included in the SANRAL (2013) Drainage Manual.

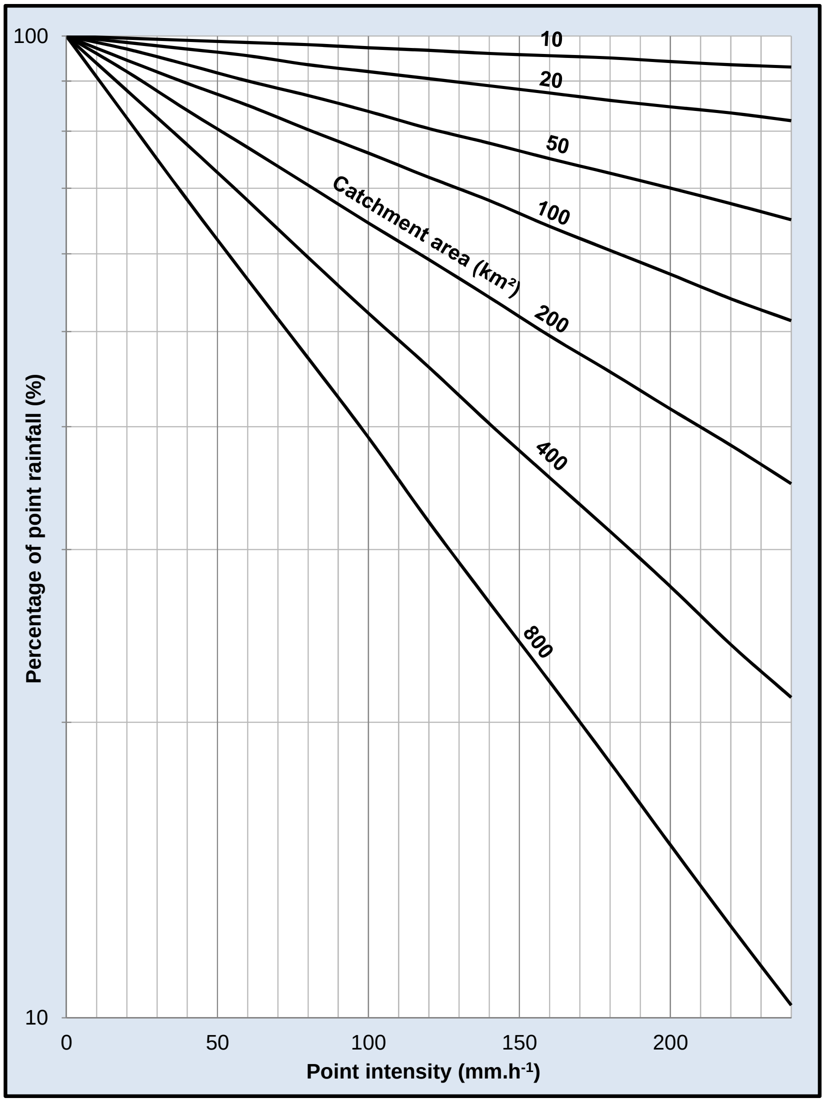

Figure 2.4: Expected percentage of runoff as a function of point rainfall intensity (SANRAL, 2013)

Op ten Noort and Stephenson (1982) converted Figure 2.4 into a mathematical expression using regression analysis. This equation is seen below as Eq. (2.5). In small catchment areas (≤ 800 km²), the ARF is mainly a function of the area and design point rainfall intensity since the relationship between rainfall intensity and the infiltration rate of the soil is predominant (Alexander, 2001; SANRAL, 2013).

$$ARF = Exp\left(-0.000068\, iA^{0.77}\right) \tag{2.5}$$

Where:

ARF = areal reduction factor for point rainfall (%),

A = catchment area (km²), and

i = point rainfall intensity at the storm centre ($mm.h^{-1}$).

(b) **Wiederhold's method (1969):** In the late 1960s, Wiederhold (1969, cited by Lambourne and Stephenson, 1986) used a variable location, storm-centred approach which is a modified version of Van Wyk's (1965) method to establish ARFs for 170 storms over large catchment areas between 500 km² and 30 000 km² within 18 regions delineated for South Africa. In these medium to large catchment areas ($A \leq 30\ 000$ km²) the ARF is mainly a function of the area and storm duration since the quantity of rainfall relative to the number of storage areas is of great importance (Alexander, 2001; SANRAL, 2013). The large area storms were delineated while the point rainfall depths at each rainfall station were used to fit a sixth-degree polynomial surface to enable the plotting of isohyets. Regionalised depth-area curves were produced for each storm at a daily interval resulting in co-axial diagrams to estimate the rainfall equalled or exceeded for storm durations of one day or longer. The developed depth-duration-area envelope diagram is shown in Figure 2.5.

In the case of large area storms with associated storm durations less than 24 hours, the areal average rainfall over increasing areas (durations of 1 to 6 days) within each of the 18 regions were expressed as percentages of the maximum observed point rainfall. Depth-area diagrams were produced for durations of 1 to 6 days. The upper envelope diagrams (of individual durations) were then re-plotted to produce depth-duration-area diagrams. Thereafter, the 24 hour to 1 hour durations were linearly extrapolated to express the rainfall associated with a given area as a proportion of the point rainfall between one and 72 hours (Lambourne and Stephenson, 1986).

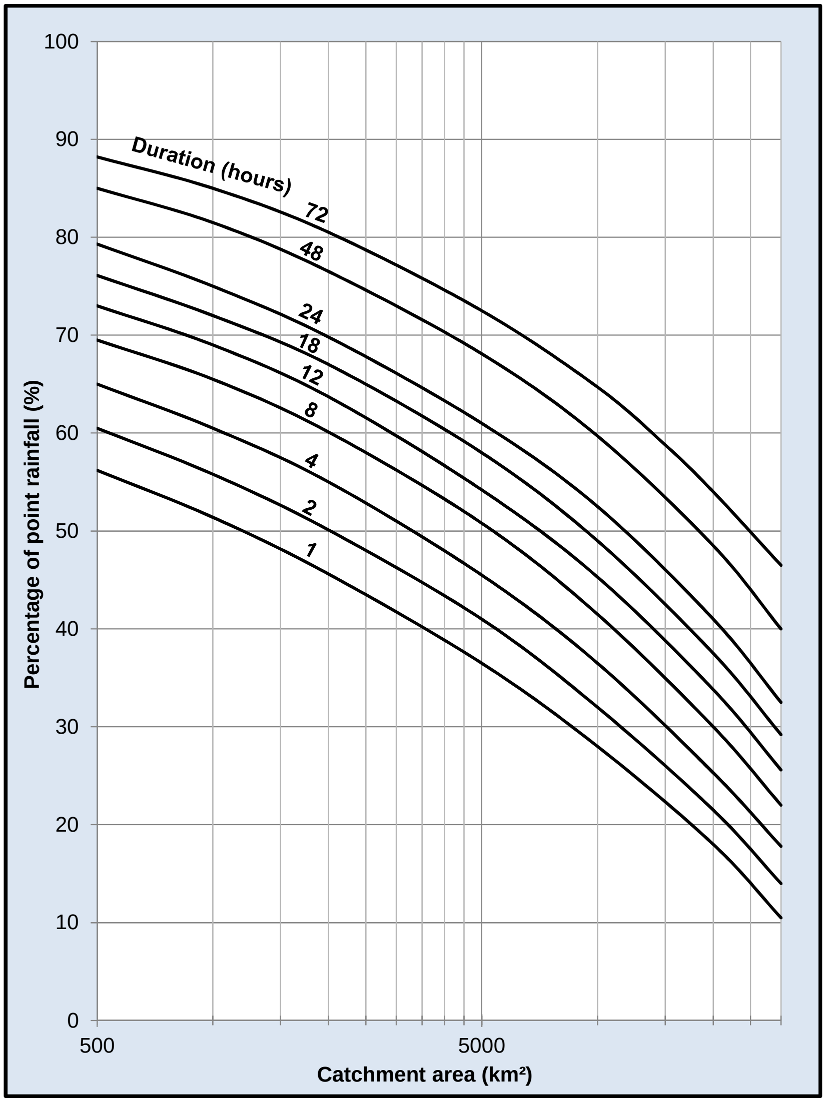

Figure 2.5: Expected percentage of runoff as a function of storm duration (SANRAL, 2013)

Op ten Noort and Stephenson (1982) converted Figure 2.5 to the mathematical expression using regression analysis as shown in Eq. (2.6).

$$ARF = \left[1.343 - 0.09Ln(A)\right] T_d^{\,0.03A^{0.19}} \tag{2.6}$$

Where:

ARF = areal reduction factor for point intensity (%),

A = catchment area (km²), and

T_d = storm duration (hours).

Op ten Noort and Stephenson (1982) compared Eqs. (2.5) and (2.6) and established that the use thereof could cause a discontinuity in storm runoff estimation. Consequently, Figure 2.5 was extrapolated such that the ARFs approach unity at short durations. This relationship is expressed by Eq. (2.7).

$$ARF = \left[1.04 - 0.08Ln(A)\right] T_d^{\,0.02A^{0.28}} \qquad (2.7)$$

Where:

ARF = areal reduction factor for point intensity (%),

A = catchment area (km²), and

T_d = storm duration (hours).

(c) **Alexander's method (1980; 2001):** Alexander (1980) developed a geographically-centred ARF relationship based on the ARF diagrams contained in the UK FSR (Figure 2.6; NERC, 1975). These ARF diagrams had an adjustment made to account for short duration rainfall over small catchment areas, which are mostly characterised by severe storm mechanisms producing very high intensity rainfall with cell core areas exceeding 10 km^2 and durations exceeding 10 minutes.

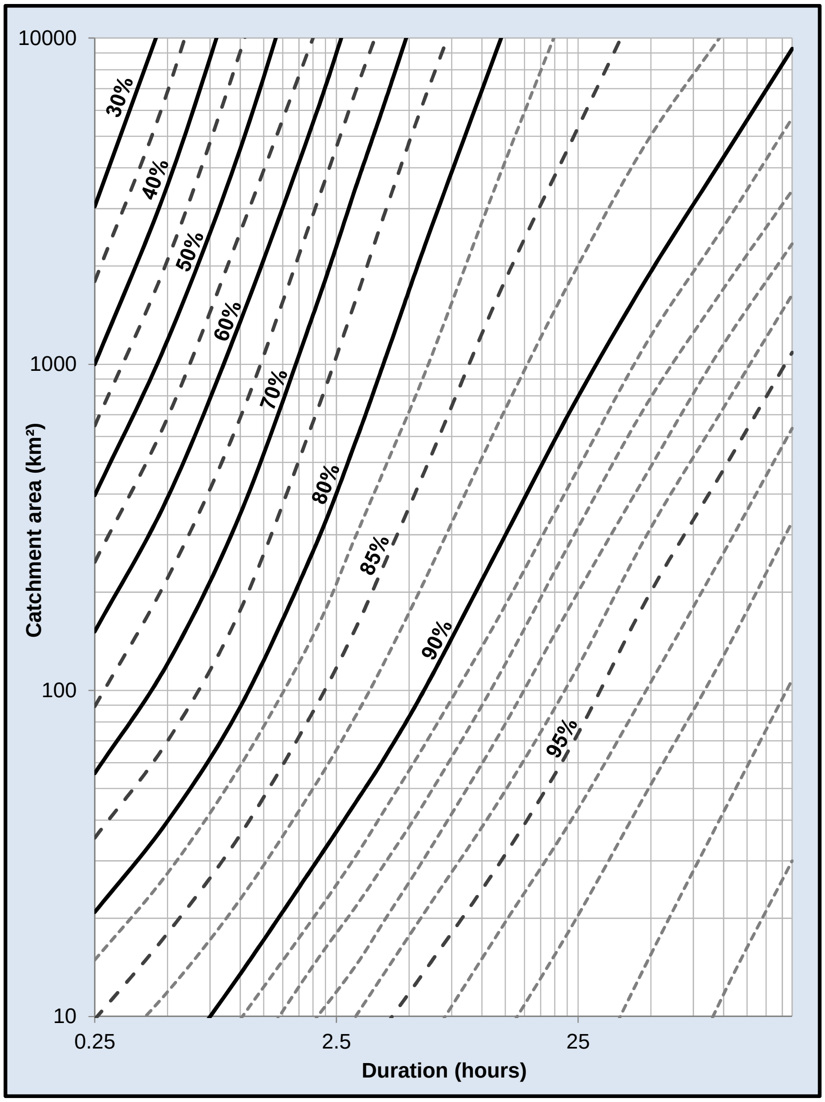

Figure 2.6: UK FSR ARF diagram (NERC, 1975)

Estimates of shorter duration rainfall based on extrapolation from longer durations are unreliable when viewed in the light of the storm mechanisms which produce high-intensity rainfall for durations less than 10 minutes (Alexander, 1980). Thus, there is little justification in assuming ARFs less than 100% in these area and duration regions; consequently the UK FSR values were adjusted accordingly (Figure 2.7).

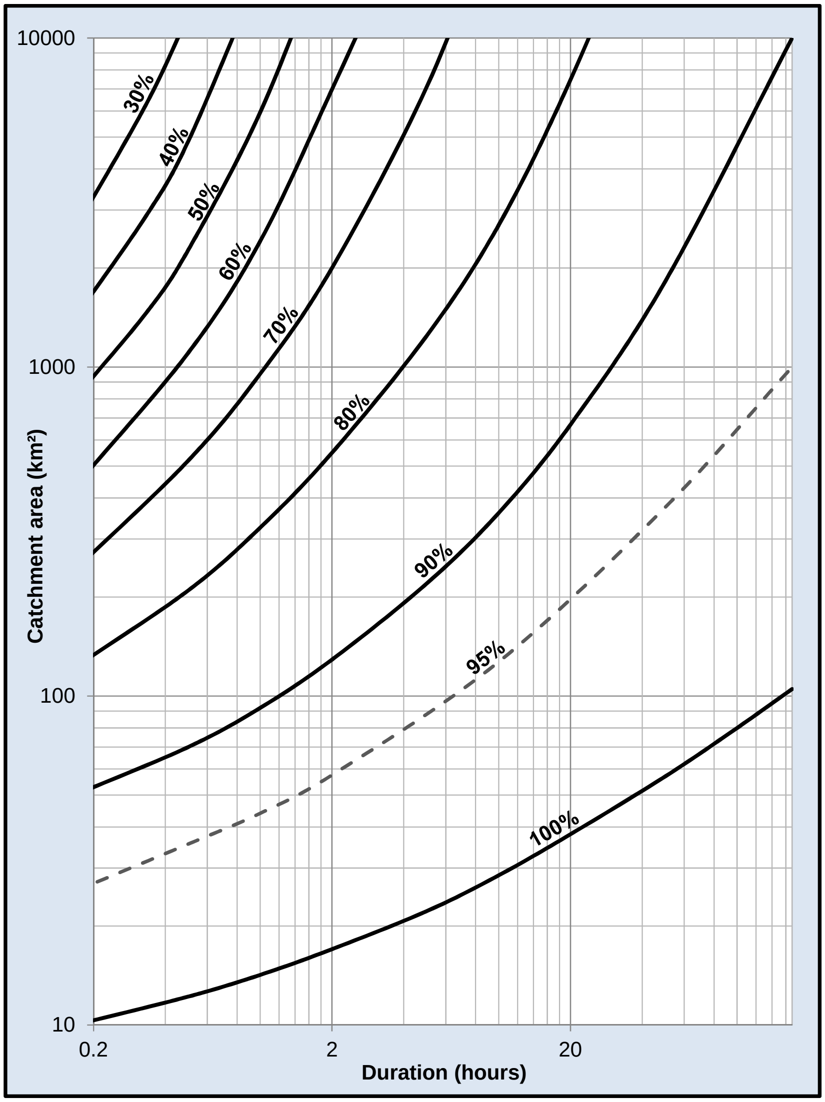

Figure 2.7: Adopted UK FSR ARFs for South Africa (after Alexander, 1980)

Alexander (2001) revised the ARF diagram (Figure 2.7) to a more reliable and user-friendly diagram that is currently used by practitioners (SANRAL, 2013). This is shown in Figure 2.8.

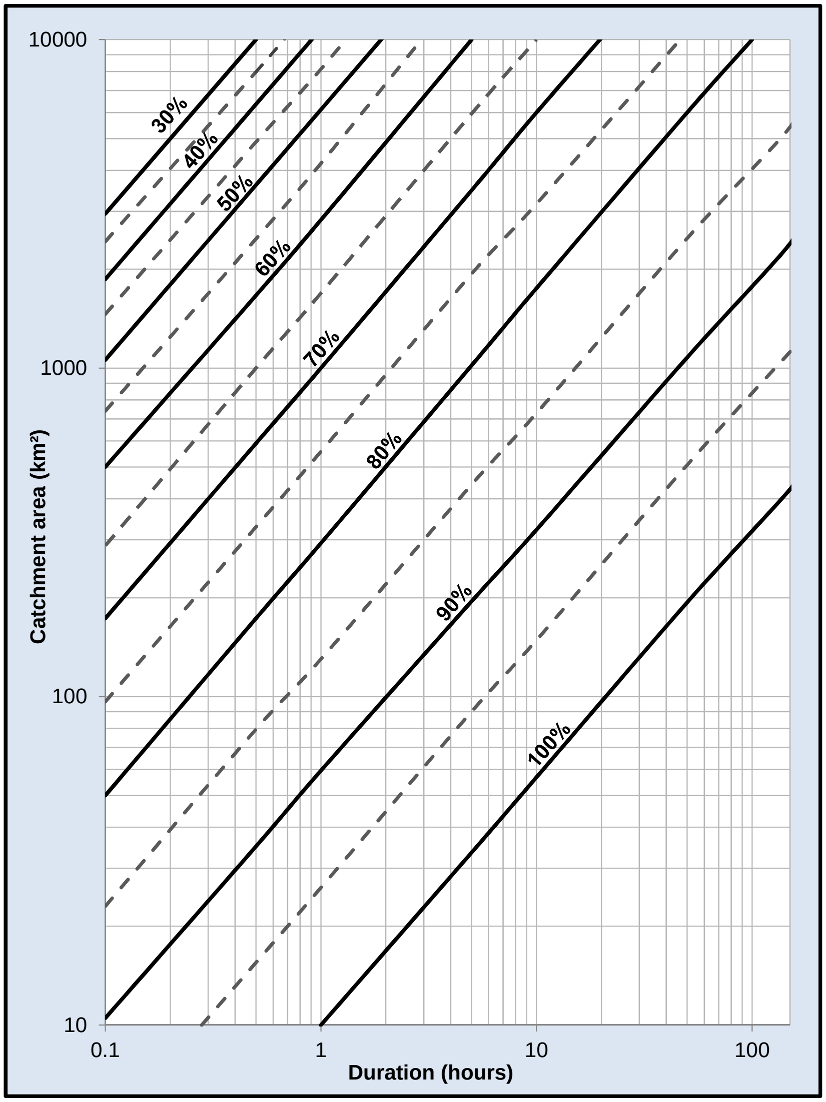

Figure 2.8: Revised ARF diagram for South Africa (Alexander, 2001)

Op ten Noort and Stephenson (1982) converted Figure 2.7 to the mathematical expression Eq. (2.8) using regression analysis. Alexander (2001) expressed Figure 2.8 in the form of a mathematical relationship as seen in Eq. (2.8a). The use of both Eq. (2.8) and (2.8a) resulted in slightly more conservative results when compared to the UK FSR and USWB values. Alexander (2001) recommended that the ARF relationship shown in Eq. (2.8a) be used for Southern African conditions where the average rainfall depth over a catchment has to be established from point rainfall statistics.

$$ARF = [1.306 - 0.0902\,Ln(A)] + Ln(T_d)[0.0161 Ln(A) - 0.0498] \quad (2.8)$$

$$ARF = [90\,000 - 12\,800\,Ln(A) + 9\,830\,Ln(60T_C)]^{0.4} \quad (2.8a)$$

Where:

ARF = areal reduction factor (%),

A = catchment area (km²),

T_C = time of concentration/critical storm duration (hours), and

T_d = storm duration (hours).

Gericke and Du Plessis (2011) established that a relationship existed between the A, T_C and ARFs. The validity of Eq. (2.8a) was assessed by plotting T_C within each catchment under consideration against the catchment area, after which a power-law curve fitted through the data points was superimposed on Figure 2.8 and the original ARF diagram as published in the UK FSR (NERC, 1975). The fitted power-law relationship was expressed as Eq. (2.9), which provides a good indication of T_C associated with any catchment area under consideration. Equation (2.10) resulted from the substitution and simplification of Eq. (2.9) into Equation (2.8a).

$$T_C = 0.2284A^{0.596} \quad (2.9)$$

$$ARF = [-6\,944.3Ln(A) + 115\,731.9]^{0.4} \quad (2.10)$$

Where:

ARF = areal reduction factor (%),

A = catchment area (km²), and

T_C = time of concentration (hours).

The results obtained from this study clearly indicate that the power-law curve yielded a constant ARF range of between 87% and 88% across the original UK FSR ARF diagram for durations exceeding three hours. Gericke and Du Plessis (2011) concluded that the ARF relationship expressed by Eq. (2.10) can be used instead of Eq. (2.8a) to convert design point rainfall depths or intensities to an average areal design rainfall depth or intensity.

2.7.2 International methods

Bell (1976) conducted probabilistic rainfall analyses at single rainfall stations (14 year record length) situated in circular catchment areas of 1 000 km² each in the UK to estimate areal and average design point rainfall frequency curves and to estimate ARFs. The ARFs were expressed as the ratio of areal to average point rainfall with associated AEPs.

A modified Thiessen weighting procedure was used to estimate the daily areal rainfall values, after which these values were ranked to obtain the 20 highest independent values for each sample area (Bell, 1976). In other words, a PDS using equally ranked observations curtailed to a common base period, were used and fitted to an exponential distribution with parameters estimated by the Method of Maximum Likelihood (MML). The average design point rainfall frequency curves were estimated using the 20 highest daily rainfall values at each rainfall station (Bell, 1976). Instead of deriving separate frequency curves for each rainfall station to estimate weighted averages, a simpler equivalent procedure was adopted. Each ranked weighted average point rainfall value was determined using the same modified Thiessen weighting procedure, followed by the exponential distribution curve fitting to provide estimates of the average design point rainfalls for recurrence intervals from 2 to 20 years. The ARFs were then estimated directly using the corresponding areal and average design point rainfall values associated with each recurrence interval or AEP (Siriwardena and Weinmann, 1996).

Bell (1976) concluded that this procedure is probabilistically more correct due to the inclusion of AEPs, while the derived ARFs based on daily (24-hour) and sub-daily rainfall (1-hour to 2-hours), proved to vary between 5% and 10% respectively. The ARF estimates also compared reasonably with the 2 to 20-year recurrence interval ARF estimates contained in the UK FSR (NERC, 1975) while, for the higher recurrence intervals (50- to 100-year), the ARF estimates were significantly lower (Siriwardena and Weinmann, 1996). The mathematical relationship representative of Bell's (1976) method is shown in Eq. (2.11).

$$ARF_m = \frac{\sum_{i=1}^{N} \left(w_i \overline{\overline{P_{ij}}} \right)_m}{\sum_{i=1}^{N} \left(w_i P_{ij} \right)_m} \qquad (2.11)$$

Where:

ARF_m = areal reduction factor [ratio of the areal rainfall of rank *m* to the Thiessen weighted average point rainfall of the same rank (%)],

m = rank value

N = number of rainfall stations within the catchment area,

$\overline{\overline{P_{ij}}}$ = point rainfall for station *i* on the day of the annual maximum areal rainfall in year *j* (mm),

P_{ij} = annual maximum point rainfall of station *i* in year *j* (mm), and

w_i = ratio of the areal rainfall of rank *m* to the Thiessen weighted average point rainfall of the same rank.

The ARF estimation methodology proposed by Bell (1976), is not only probabilistically more correct than the USWB and UK FSR ARF estimation methods, but the variation of ARFs with recurrence interval is also clearly evident when ARFs are directly obtained from the areal and point design rainfall frequency curves. In most other ARF estimation methods, *e.g.* USWB and UK FSR, the variation of ARFs are largely obscured by the regionalisation of data.

Apart from the South African methods and Bell's (1976) method discussed in this chapter, Tables A1 and A2 in Appendix A contain a summary of the empirical and analytical ARF estimation methods used internationally.

An overview of the location and characteristics of the pilot study area (C5 secondary drainage region) is provided in the next chapter.

CHAPTER 3 : STUDY AREA

This chapter provides an overview of the location and characteristics of the pilot study area (C5 secondary drainage region).

3.1 Location and General Characteristics

South Africa is demarcated into 22 primary drainage regions, which are further delineated into 148 secondary drainage regions. The pilot study area is situated in the C5 secondary drainage region within the primary drainage region C (Midgley *et al.*, 1994). The pilot study area covers 34 795 km^2 and, as shown in Figure 3.1, is located between 28°25' and 30°17' S and 23°49' and 27°00' E (DWAF, 1995).

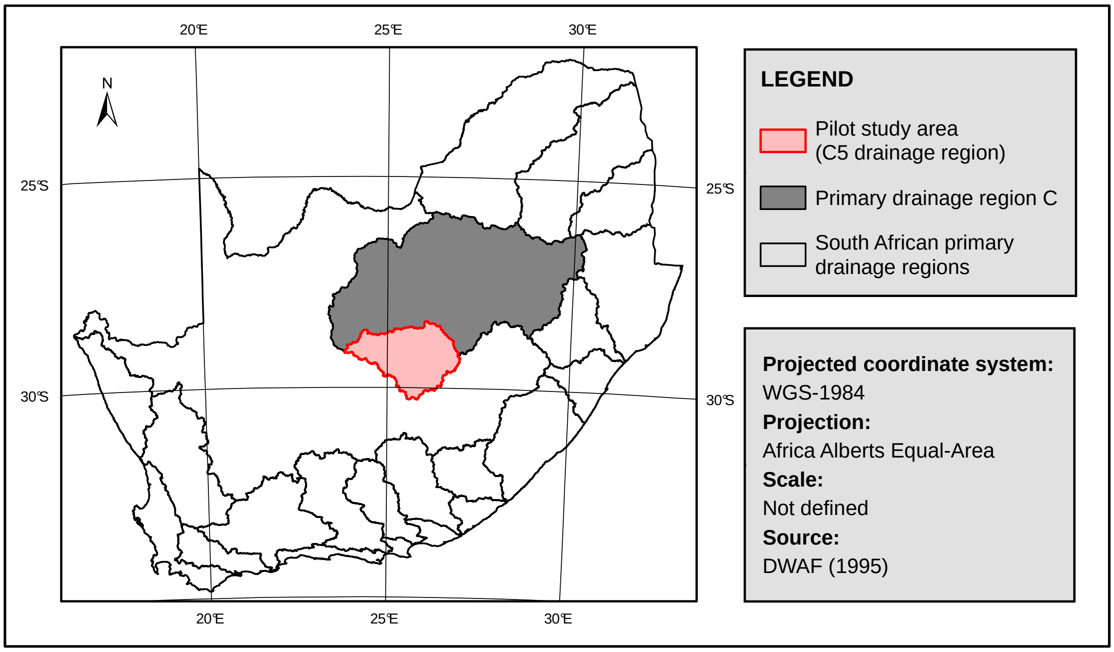

Figure 3.1: Location of the pilot study area (Gericke and Smithers, 2014)

Two tertiary drainage regions, namely, the Riet River Catchment (RRC), also known as C51, and the Modder River Catchment (MRC), also known as C52, are located in the pilot study area and cover an area of 17 435 km^2 and 17 360 km^2 respectively (Midgley *et al.*, 1994). The RRC consists of twelve QCs, whereas the MRC consists of eleven QCs. The Modder and Riet Rivers are the main river reaches and discharge into the Orange-Vaal River drainage system (Midgley *et al.*, 1994).

The area ranges of each QC are listed in Table 3.1.

Table 3.1: Quaternary catchments within the C5 secondary drainage region

QC	Area (km^2)	Cover (%)
C51A	675	1.9
C51B	1 691	4.9
C51C	624	1.8
C51D	922	2.6
C51E	806	2.3
C51F	876	2.5
C51G	1 835	5.3
C51H	1 781	5.1
C51J	1 051	3.0
C51K	3 628	10.4
C51L	2 029	5.8
C51M	1 518	4.4
C52A	937	2.7
C52B	949	2.7
C52C	600	1.7
C52D	471	1.4
C52E	897	2.6
C52F	688	2.0
C52G	1 789	5.1
C52H	2 373	6.8
C52J	1 922	5.5
C52K	4 331	12.4
C52L	2 404	6.9
Total	**34 795**	**100.0**

The natural vegetation is dominated by the grasslands of the Interior Plateau, False Karoo and Karoo. Cultivated land is the largest human-induced land cover alteration in the rural areas, while residential and suburban areas dominate the urban areas (CSIR, 2001). Almost 99.1% of the pilot study area consists of rural areas, while 0.7% and 0.2% represent urban areas and water bodies respectively (DWAF, 1995).

The topography is gentle with elevations varying from 1 021 m to 2 120 m and average catchment slopes ranging between 1.7% and 10.3% (USGS, 2002). The catchment slope distribution of the pilot study area is summarised in Table 3.2 and is based on the reclassified Shuttle Radar Topography Mission (SRTM) elevation data for Southern Africa at 90-metre resolution (USGS, 2002).

Table 3.2: Catchment slope distribution (USGS, 2002)

Slope classification (%)	Area (km²)	%-Distribution
0 – 3	21 921	63
3 – 10	10 786	31
10 – 30	1 740	5
> 30	348	1
Total	**34 795**	**100**

3.2 Climate

The climate of central South Africa is moderate to hot in the summer. The average long-term minimum and maximum temperatures in the summer vary between 12℃ and 30℃, while the winter months are characterised by average long-term minimum and maximum temperatures ranging between 3°C and 18℃ (Midgley *et al.*, 1994). The Mean Annual Evaporation (MAE) varies from 1 600 mm (where the Modder River originates) to 2 200 mm (downstream of the confluence of the Modder and Riet Rivers). Evaporation increases from the east to the west as opposed to the rainfall which decreases from east to west (Midgley *et al.*, 1994).

The average MAP for the C5 secondary drainage region is 424 mm, ranging from 275 mm in the west to 685 mm in the east (Lynch, 2004) and rainfall is characterised as highly variable and unpredictable. The rainy season starts in early September and ends in mid-April with a dry winter. The rainfall intensity in this region is normally high to very high with associated thunder activity and can be classified as convective rainfall (Lynch, 2004).

3.3 Rainfall Monitoring Network

The 185 SAWS daily rainfall stations located within the boundaries of the C5 secondary drainage region are shown in Figure 3.2. It is evident from Figure 3.2 that the rainfall monitoring network is in general denser in the mid-eastern parts of the study area as opposed to the north-western parts. The overall distribution and location of the individual rainfall stations are regarded as sufficient for the purpose of this study. However, when point rainfall depths are converted to rainfall depths over an area using averaging techniques, such as the Thiessen polygons, denser rainfall monitoring networks are preferred. Therefore, the 38 neighbouring rainfall stations of C5 secondary drainage region are also considered in this study and are shown in Figure 3.2. This results in a total of 223 rainfall stations. The rainfall stations are almost evenly distributed across the C51 and C52 tertiary drainage regions with 123 and 100 SAWS daily rainfall stations respectively.

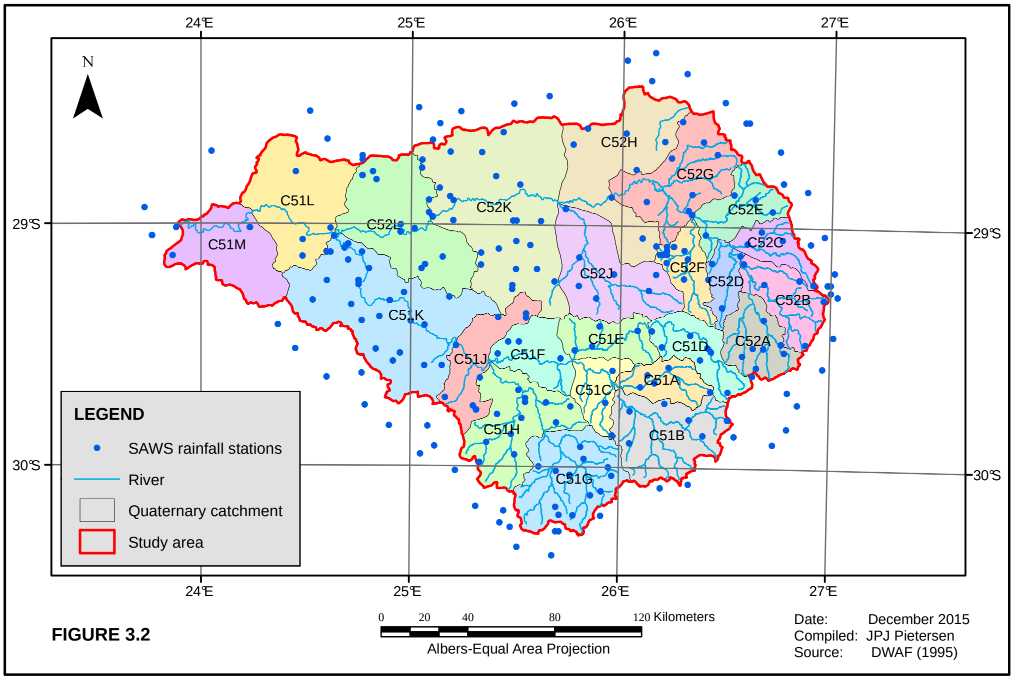

Figure 3.2: Location of the daily SAWS rainfall stations in the pilot study area

The details of each daily rainfall station in terms of location within each QC and record length (RL) in years are listed in Table A.3, Appendix A. It is evident from Table A.3 that some rainfall stations have limited record lengths, and the period of record within a specific QC does not necessarily agree or overlap. Due to insufficient overlapping of rainfall station recordings, as well as the basic requirement of the averaging techniques such as the Thiessen polygons for synchronised time intervals, the 223 rainfall stations identified need to be evaluated and filtered prior to any analyses conducted to derive representative ARF values.

Such a filtering or selection procedure is explained and expanded in the next chapter. The methodologies adopted in meeting the specific objectives of this study are also discussed in the next chapter.

CHAPTER 4 : METHODOLOGY

This chapter presents the methodology followed to estimate geographically-centred ARFs representative of the different rainfall-producing mechanisms at a QC level in the C5 secondary drainage region. The procedural steps followed to derive probabilistically correct ARFs are discussed in Sections 4.1 to 4.5, while the comparison and assessment of the derived ARFs to a selection of ARF methods currently used both in South Africa and internationally are discussed in Section 4.6.

4.1 Analysis of Rainfall Data

A daily rainfall database was established by evaluating, preparing and extracting daily rainfall data from the SAWS rainfall stations present in the C5 secondary drainage region as well the data from neighbouring rainfall stations. The Geographical Information System (GIS) feature classes (shape files) containing the spatial features of the complete daily rainfall database were generated in the ArcGIS^{TM} 10.1 environment. The daily rainfall database typically consists of up-to-date, aggregated QC-specific samples of autographic daily and sub-daily (where available) rainfall stations as previously used by Smithers and Schulze (2000a; 2000b) and Lynch (2004).

The minimum number of rainfall stations in each QC was based on the criteria recommended by Siriwardena and Weinmann (1996), namely, a minimum of three rainfall stations for catchment areas up to 500 km^2 plus one additional station for every 500 km^2 thereafter. The geographical location of each rainfall station within a particular QC was also used to determine the relative importance of each station. For instance, rainfall stations within a QC boundary are ranked as superior to surrounding neighbouring rainfall station(s) when point rainfall depths are averaged over an area to provide areal rainfall depths.

The DREU (Lynch, 2004), as described in Chapter 2, was used for the extraction and infilling of all the daily rainfall data series. Each rainfall station identified with the DREU was evaluated in terms of record length (≥ 30 years), data quality and geographical location in relation to the specific QC under consideration. Thereafter, the daily point rainfall AMS and areal AMS were established and extracted for the purpose of probabilistic analyses.

The areal AMS was estimated by multiplying each rainfall station's 1-day rainfall value within a particular QC by the corresponding areal weight on a daily basis. The daily point and areal rainfall series recorded at fixed 24-hour intervals were converted to a continuous 24-hour rainfall series by considering two different methods, namely, the use of conversion factors (Adamson, 1981) and scaling factors (Smithers and Schulze, 2003). The two methods were compared and the more robust method (based on statistical properties) was then used for the derivation of probabilistically correct ARFs.

4.1.1 Infilling of missing rainfall data

The DREU (Lynch, 2004) was used to determine the best approach to infill the missing data for the rainfall station(s) under consideration. Infilling was not only used to extend the rainfall data series at particular rainfall station(s) but also to ensure that there is sufficient synchronisation and overlapping between the various rainfall station recordings to extract a complete 1-day areal AMS within a specific QC. The infilling of missing data was carefully done at a daily time interval to result in the most comprehensive record length, *i.e.* the longest possible rainfall record with mutual starting and ending dates to ultimately enable the extraction of the catchment AMS. In each QC, the infilling procedure resulted in an AMS with at least 60 years of daily rainfall data.

4.1.2 Conversion factors

Each daily observed point rainfall data series recorded at a fixed 1-day interval was converted to a continuous 24-hour rainfall series. The conversion factors, as proposed by Adamson (1981) and contained in Table 2.1 (refer to Chapter 2), were used to convert the 24-hour continuous rainfall to durations ranging between 1 hour and ≤ 24 hours. In the case of durations longer than one day, the accumulated daily rainfall totals over a period of two to seven days were estimated by applying a 'sliding window' approach to the 1-day observed rainfall data series.

For example, the accumulated daily rainfall over three days is estimated by applying a '3-day sliding window' approach to the 1-day observed rainfall data series. In other words, the 1-day AMS for each rainfall station was determined by extracting the maximum 1-day rainfall values within each hydrological year for the complete infilled record length. Thereafter, a 3-day window was slided over the observed rainfall 1-day data series for each hydrological year to provide the accumulated 3-day totals. The highest accumulated value within each hydrological year was then used as the 3-day AMS value. This process was then repeated for each hydrological year to result in a complete 3-day AMS at a particular rainfall station. The same procedure was performed to obtain the 7-day AMS for each rainfall station. The conversion of a fixed interval to a continuous measurement (for example, a 1-day interval to 24 hours or 3-day interval to 72 hours) was based on the specific conversion factors as listed in Table 2.2 (refer to Chapter 2).

4.1.3 Scaling factors

Each daily observed point rainfall data series recorded at a fixed 1-day interval was converted to a continuous 24-hour rainfall series by making use of the scaling factors as proposed by Smithers and Schulze (2003). The latter scaling factors are based on the mean of the 1-day AMS. The method adopted to derive the appropriate scaling factors in each QC may be summarised as follows:

(a) The mean values of the 1-day AMS values for both the short and long duration cluster groups (Smithers and Schulze, 2003) are retrieved for each rainfall station.

(b) The 24-hour AMS mean values are estimated by using the ratios in Table 4.1.

(c) The short duration ($\leq$ 24-hour) AMS mean values (1-hour, 8-hour and 16-hour) for each rainfall station are estimated using the regression parameters and statistics as listed in Table 4.2 and expressed in Eq. (4.1) (Smithers and Schulze, 2003). The cluster group is dependent on the region applicable to each rainfall station.

Table 4.1: Ratios of 24-hour: 1-day AMS mean values (Smithers and Schulze, 2003)

Cluster	Average	Median	Std. error	Cluster	Average	Median	Std. error
1	1.20	1.20	0.049	9	1.26	1.27	0.111
2	1.21	1.21	0.063	10	1.19	1.18	0.090
3	1.19	1.18	0.072	11	1.20	1.15	0.087
4	1.21	1.22	0.090	12	1.19	1.18	0.044
5	1.20	1.17	0.097	13	1.28	1.30	0.139
6	1.17	1.16	0.055	14	1.24	1.24	0.056
7	1.15	1.14	0.051	15	1.25	1.26	0.096
8	1.20	1.20	0.032				

Table 4.2: Short duration regression parameters and statistics (Smithers and Schulze, 2003)

Duration (minutes)	Cluster Group					
	Cluster 1		Cluster 2		Cluster 3	
	Xcoeff	Constant	Xcoeff	Constant	Xcoeff	Constant
5	0.0923	3.8797	-0.0150	10.5849	0.1483	2.3719
10	0.1463	5.0415	0.0139	13.8042	0.1267	8.2786
15	0.1764	7.0258	0.0391	16.0588	0.1041	13.3526
30	0.1718	14.1536	0.0462	24.3809	0.0793	22.0109
45	0.1862	16.9113	0.0923	28.7435	0.1645	20.5722
60	0.2330	16.4947	0.1974	21.8449	0.2532	17.4304
90	0.2908	15.9214	0.3229	17.3012	0.3478	14.5765
120	0.3604	14.0952	0.4249	13.5000	0.3860	14.1375
240	0.5041	11.7955	0.6272	7.0738	0.4859	12.9281
360	0.5769	10.9397	0.7310	3.9253	0.5578	11.4020
480	0.6441	9.6918	0.8503	-0.3842	0.6152	9.8269
600	0.7130	7.5810	0.8495	1.3596	0.6513	9.4183
720	0.7553	6.4667	0.8821	0.7691	0.6901	8.5148
960	0.8468	3.9779	0.9205	0.5318	0.7794	5.6472
1200	0.9464	0.7498	0.9678	-0.3519	0.8865	2.4208
Duration (minutes)	**Cluster 4**		**Cluster 5**		**Cluster 6**	
	Xcoeff	**Constant**	**Xcoeff**	**Constant**	**Xcoeff**	**Constant**
5	0.1143	2.5965	0.0024	8.7465	0.0396	3.3994
10	0.2097	2.4553	0.0254	11.5868	0.0422	5.4591
15	0.2584	3.1253	0.0869	11.9385	0.0472	6.7972
30	0.3403	4.2770	0.2075	12.8711	0.0749	8.6806
45	0.4054	4.0300	0.2973	12.4460	0.1076	9.2315
60	0.4470	4.0338	0.3845	10.6553	0.1321	9.7558
90	0.4966	3.7201	0.4204	12.4383	0.1896	9.8328
120	0.5501	2.8506	0.4507	13.0382	0.2401	9.6347
240	0.5875	4.2623	0.6587	6.9769	0.3832	8.8665
360	0.6640	3.3691	0.7597	4.0615	0.5093	7.0885
480	0.7214	2.7333	0.8613	0.4021	0.5807	6.5744
600	0.7725	2.2127	0.9577	-3.3262	0.6495	5.4221
720	0.8188	1.6941	0.9687	-2.7026	0.7152	4.3546
960	0.9299	-0.4875	0.9737	-1.3786	0.8340	2.1099
1200	0.9300	0.9739	1.0094	-1.9000	0.9219	0.7993

Table 4.2: (continued)

Duration (minutes)	Cluster Group					
	Cluster 7		Cluster 8		Cluster 9	
	Xcoeff	Constant	Xcoeff	Constant	Xcoeff	Constant
5	0.0519	6.3133	0.0517	4.5662	-0.0072	5.9777
10	0.0565	10.0239	0.0834	6.9401	0.0045	7.5117
15	0.0681	12.5861	0.1169	7.7585	0.0159	8.6239
30	0.0821	19.0440	0.1326	15.0990	0.0241	11.7661
45	0.1037	22.0067	0.1224	21.9088	0.0488	12.7010
60	0.1109	25.1194	0.1547	22.9352	0.0941	12.0246
90	0.1521	26.7598	0.2163	22.4071	0.1445	12.0904
120	0.2024	26.4768	0.2593	22.9463	0.1968	12.1137
240	0.3626	22.0900	0.3449	26.5535	0.3867	9.5118
360	0.5117	15.8893	0.4139	27.5278	0.5117	7.9027
480	0.6301	9.8498	0.4899	25.3092	0.6023	6.6963
600	0.7139	7.0232	0.5535	24.6865	0.6942	4.7192
720	0.7672	6.1739	0.6103	23.8021	0.7520	3.8480
960	0.8655	3.7845	0.8220	7.9328	0.8479	2.7507
1200	0.9348	1.5801	0.9665	-1.6203	0.9280	13.8280
Duration (minutes)	**Cluster 10**		**Cluster 11**		**Cluster 12**	
	Xcoeff	**Constant**	**Xcoeff**	**Constant**	**Xcoeff**	**Constant**
5	0.0084	6.1800	0.0669	5.0539	0.1295	1.9085
10	0.0068	9.1578	0.1200	6.3158	0.1611	3.7402
15	-0.0069	11.8186	0.0855	11.3601	0.1940	4.8688
30	0.0273	14.2721	0.0811	17.4901	0.2707	5.7881
45	0.0595	14.7732	0.1226	18.5496	0.3439	4.6457
60	0.0950	14.6028	0.1424	19.6956	0.3992	3.6599
90	0.1445	14.4951	0.2301	17.6776	0.4440	3.5675
120	0.1927	14.0599	0.2719	17.5168	0.4707	4.0248
240	0.3616	10.8995	0.4109	14.9130	0.5777	3.7441
360	0.4650	9.3312	0.4967	13.2305	0.6113	5.1719
480	0.5456	7.9258	0.6113	9.3418	0.7178	2.5954
600	0.6514	5.3833	0.6829	7.5003	0.7383	3.1730
720	0.7107	4.9766	0.7676	4.6418	0.7625	3.5743
960	0.8068	3.4508	0.8707	1.9546	0.8091	3.9721
1200	0.9473	0.0995	0.9697	-0.7085	0.8781	3.0684
Duration (minutes)	**Cluster 13**		**Cluster 14**		**Cluster 15**	
	Xcoeff	**Constant**	**Xcoeff**	**Constant**	**Xcoeff**	**Constant**
5	-0.0226	9.1244	0.0483	5.6177	0.1634	-0.2312
10	-0.0037	11.6905	0.0894	7.1337	0.2447	-0.6720
15	-0.0054	14.8076	0.1095	9.4261	0.3055	-0.9936
30	0.0169	18.4250	0.2516	7.8713	0.4126	-1.4910
45	0.0552	18.6612	0.3765	4.5577	0.4872	-1.9825
60	0.0919	18.2126	0.4444	2.8843	0.5158	-1.7239
90	0.1582	16.6326	0.5433	0.6221	0.5851	-1.8885
120	0.2257	14.7643	0.5944	0.0660	0.6286	-1.8933
240	0.3930	11.1904	0.7330	-1.6145	0.7110	-1.2663
360	0.5268	8.0679	0.8171	-2.7460	0.7895	-1.4310
480	0.6180	5.9437	0.8654	-2.8754	0.8236	-1.0729
600	0.7121	3.1372	0.9558	-5.5491	0.8529	-0.8552
720	0.7772	1.8094	0.9367	-3.3439	0.8906	-0.8888
960	0.8753	0.0716	0.9003	0.6960	0.9153	-0.0197
1200	0.9626	-0.9622	0.9970	-2.1296	0.9532	0.3203

The short duration (≤ 24-hour) AMS mean values (1-hour, 8-hour and 16-hour) for each rainfall station are expressed using Eq. (4.1) (Smithers and Schulze, 2003).

$$L\_24_D = Const + Xcoeff \ * L\_1_{24} \quad (4.1)$$

Where:

L_24_D = AMS mean value for any duration (D) < 24-hour (mm),

L_1_{24} = AMS mean value for 24-hour (mm),

Const = regression constant, and

Xcoeff = regression coefficient.

(d) The long duration (> 24-hour) AMS mean values for each rainfall station are estimated using the regression parameters and statistics as listed in Table 4.3 in conjunction with Eqs. (4.2), (4.3) and (4.4) (Smithers and Schulze, 2003).

$$L\_1_D = \phi_D + \left(\alpha_D * L\_1_{1day}\right) \quad (4.2)$$

$$\alpha_D = \theta + \tau * D^{\sigma} \quad (4.3)$$

$$\phi_D = v + k * D^{\rho} \quad (4.4)$$

Where:

L_1_D = AMS mean value for any duration (D) > 24-hour (mm),

α_D = regression coefficient for any duration (D) > 24-hour,

ϕ_D = regression constant for any duration (D) > 24-hour,

L_1_{1day} = AMS mean value for 1-day (mm),

θ = regression constant,

τ = regression coefficient,

σ = transformation exponent for any duration (D) > 24-hour,

v = regression constant,

k = regression coefficient, and

ρ = transformation exponent for any duration (D) > 24-hour.

Table 4.3: Long duration regression parameters and statistics (Smithers and Schulze, 2003)

Variable	Regression statistics	Region (Number of stations)						
		1 (596)	2 (173)	3 (62)	4 (264)	5 (234)	6 (401)	7 (75)
α_D	θ	0.60	3.37	5.10	0.15	2.20	-0.02	-0.86
	τ	0.39	-2.31	-4.04	0.90	-1.16	1.02	1.87
	σ	0.68	-0.19	-0.12	0.42	-0.41	0.35	0.27
ϕ_D	k	-2.16	47.11	23.60	-11.53	0.49	-9.68	-17.78
	v	6.09	-46.38	-22.54	11.01	-3.48	11.92	18.96
	ρ	1.21	-0.14	-0.36	0.56	1.68	0.54	0.33

(e) The final scaling factors are obtained by dividing the derived AMS mean values for durations of 1, 8, 16, 24, 72 and 168 hours by the mean 1-day AMS value (Smithers and Schulze, 2003).

(f) A comprehensive summary table for each QC is generated. The latter tables contain all the relevant information pertaining to the rainfall stations and associated durations. Therefore, by considering the various durations, the mean is estimated for each QC thus ensuring that each QC has a defined set of scaling factors that could be applied to the fixed 1-day AMS.

(g) Finally, the derived scaling factors are used to convert the fixed 24-hour interval AMS to continuous AMS values applicable to various durations.

4.2 Averaging of Observed Rainfall

All the various methods proposed for the averaging of point rainfall depths over an area (refer to Chapter 2, Section 2.4) were considered in this study. However, Gericke and Du Plessis (2011) confirm that there is a high degree of association (r^2 values > 0.9) between the various averaging methods when applied to the C5 secondary drainage region, with percentage differences < 17%. The latter results actually confirm the even spatial distribution of the rainfall stations and the relatively flat topography of the C5 secondary drainage region (Gericke and Du Plessis, 2011). Based on these findings, the large amount of data and computations necessary, and the preferential use of the Thiessen polygon method in various international ARF studies such as Bell (1976), Stewart (1989) and

Siriwardena and Weinmann (1996), the Thiessen polygon method was selected as the most suitable method to use.

As a result, the *Areal Rain* extension in ArcView 3.2a was used to generate Thiessen polygons representative of the averaged rainfall depths for a particular area or QC under consideration. The boundary of the resultant Thiessen polygons was selected in each case, either by the applicable QCs (polygon feature classes) or by a buffered group of neighbouring rainfall stations (point feature classes). The latter option provides an alternative that allows the inclusion of rainfall stations located outside the boundary of a QC. Thiessen weights were used to estimate each rainfall station's contribution to the daily point and daily areal rainfall in each QC. In other words, the Thiessen weights were applied at a daily interval. However, rainfall stations characterised by negligible Thiessen weights at a QC level were excluded from the final analyses. As a result, two sets of weighted AMS values (point and areal) at a QC level were used during the probabilistic analyses as discussed in the next section.

4.3 Probabilistic Analyses of Weighted AMS

The quintile estimates of point and areal rainfall are more reliable when a longer period of record is used. Therefore, probabilistic analyses were conducted using an optimum infilled record length, *i.e.* the longest record length meeting all the data quality criteria. The latter optimum infilled record length was established by comparing probabilistic analysis results at 10-year ascending record length increments. The probabilistic analyses of point and areal rainfall AMS were conducted separately to result in separate point and areal design rainfall frequency curves. Probabilistic analyses of the point rainfall AMS were conducted at a QC level to result in one representative QC frequency curve condensing information from all the point rainfall data series within a particular QC.

Two different approaches were considered and compared to identify the optimum procedure to provide QC design point rainfall frequency curve(s). The steps followed in each approach are summarised below:

Approach 1 (Multiple L-moment sets):

(a) Extraction of the point rainfall AMS at each individual rainfall station;

(b) Standardisation of the point rainfall AMS at each individual rainfall station, *i.e.* the sum of the AMS values is divided by the mean of the AMS at the rainfall station under consideration;

(c) Estimation of the first three L-moments (β_0, β_1 and β_2) for each standardised rainfall series having at least 30 years of data;

(d) Estimation of weighted averages of all required L-moments (each rainfall station's contribution to the regional average is weighted in proportion to its record length by the factor N_i/L, where N_i is the record length at station i, and L is the sum of all N_i values);

(e) Probabilistic (GEV/PWM) curve fitting to the regional L-moments;

(f) Quantile estimation of regional standardised design rainfall (growth factors) based on the GEV/PWM distribution; and

(g) Site-specific quantile estimation by multiplying the standardised rainfall quantiles with the standardising value of the specific rainfall station under consideration.

Approach 2 (Single set of L-moments):

(a) Extraction of the point rainfall AMS at each individual rainfall station;

(b) Weighting of the point rainfall AMS values using Thiessen polygons to result in average point AMS values representative of all the rainfall stations in a particular QC;

(c) Standardisation of the average point rainfall AMS at a QC level, *i.e.* the sum of the average point AMS values is divided by the mean of the average point AMS values;

(d) Estimation of the first three L-moments (β_0, β_1 and β_2) for each standardised rainfall series having at least 30 years of data;

(e) Probabilistic (GEV/PWM) curve fitting to the regional L-moments; and

(f) Quantile estimation of regional standardised design rainfall (growth factors) based on the GEV/PWM distribution.

Approach 2 was used in this study, since the use of a single set of L-moments obtained from multiple rainfall stations at a QC level is preferred to the use of multiple L-moment sets obtained from individual rainfall stations.

Probabilistic analyses of the areal rainfall AMS (extracted and weighted at a daily time interval within a particular QC) were also conducted at a QC level to result in one representative areal QC frequency curve which condenses information from all the areal design rainfall data series within a particular QC. In other words, the DCR approach (as recommended by Van der Spuy and Rademeyer (2014) and discussed in Chapter 2) was used to estimate one set of daily observed areal rainfall values by applying Thiessen weights to daily point rainfall values at each rainfall station within a particular catchment. The observed daily areal rainfall AMS values were then probabilistically analysed to result in a single set of areal design rainfall values applicable to each QC under consideration.

The selection of the most suitable theoretical probability distribution was based on the statistical properties (mean, standard deviation, skewness and coefficient of variation) of each point and/or areal rainfall AMS. Typically, the LN, LP3, GEV and GLO probability distributions were considered for the frequency analyses and probabilistic curve fitting. However, Smithers and Schulze (2000a) highlighted that the GEV probability distribution is regarded as the most suitable distribution to estimate 1-day design rainfall values in South Africa. Consequently, the GEV probability distribution was regarded as statistically more robust with respect to sampling variability when ARFs or design rainfall values are estimated (Siriwardena and Weinmann, 1996; Smithers and Schulze, 2000a) and was therefore adopted in this study.

4.4 Estimation of ARFs

The estimation of ARFs was based on a modified version of Bell's (1976) method since the AMS of point and areal rainfall was used as opposed to the PDS. This modification reflects the variation of ARFs with RIs, instead of using equally ranked observations curtailed to a common base period. Sample values of the fixed-area ARFs were estimated using Approach 2 as described in Section 4.3 and expressed as the ratio between areal design rainfall (catchment) and design point rainfall (single station) estimates, in other words, design rainfall estimates corresponding to the same RI.

One set of ARFs was estimated for each QC with durations of 1, 8, 16, 24, 72 and 168 hours with corresponding RIs of 2, 5, 10, 20, 50, 100 and 200 years. The ARFs from individual QCs in the C5 secondary drainage region were pooled together to estimate mean ARFs for a combination of catchment areas, storm durations, MAP and RI values. These QC sample-mean ARF values were then used to derive functional mathematical ARF design values, which are discussed in the next section.

4.5 Derivation of ARF Algorithms

Stepwise multiple linear regression analyses were performed on the sample-mean ARF values (dependent criterion variables) and the independent predictor variables to establish calibrated ARF algorithms representative of the physiographical QC indices influencing the temporal and spatial rainfall distribution in each QC.

The following independent predictor variables were considered for inclusion:

(a) **Catchment area (*A*, km²):** The areas vary between 471 km² and 4 331 km² in the C5 secondary drainage region.

(b) **Storm duration (*D*, hours):** The selected storm durations include the 1, 8, 16, 24, 72 and 168 hour durations.

(c) **Recurrence interval (RI or *T*, years):** The selected RIs include the 2, 5, 10, 20, 50, 100 and 200 year values.

(d) **MAP (mm):** The catchment MAP varies between 326 mm and 576 mm in the C5 secondary drainage region.

Linear backward stepwise multiple regression analyses with deletion were used to remove the insignificant potential independent predictor variables (in a normal and/or transformed format) at each step to minimise the total variation, while the included independent predictor variables were tested for statistical significance at a 95% confidence level. Hypothesis testing was performed at each step to ensure that only statistically significant independent variables were retained in the model, while insignificant variables were removed. Partial *t*-tests were used to test the significance of individual independent variables, while total *F*-tests were used to determine whether an ARF as a dependent variable is significantly correlated to the independent predictor variables included in the model (McCuen, 2005). A rejected null hypothesis [*F*-statistic of observed value (*F*) > critical *F*-statistic (F_α)] was used to identify the significant contribution of one or more of the independent variables towards the prediction accuracy. The Goodness-of-Fit (GOF) statistics were assessed using the coefficient of multiple-correlation [Eq. (4.5)] and the standard error of estimate [Eq. (4.6)] (McCuen, 2005).

$$R_i^2 = \frac{\sum_{i=1}^{N}\left(y_i - \bar{x}\right)^2}{\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2} \tag{4.5}$$

$$S_{Ey} = \left[\frac{1}{v}\sum_{i=1}^{N}(y_i - x_i)^2\right]^{0.5} \tag{4.6}$$

Where:

R_i = multiple-correlation coefficient for an equation with *i* independent variables,

S_{Ey} = standard error of estimate,

i = number of independent variables,

N = number of observations (sample size),

v = degrees of freedom (*N*- *i*; with intercept = 0),

x_i = observed value (dependent variable),

$\bar{x}$ = mean of observed values (dependent variables), and

y_i = estimated value of dependent variable (x_i).

The methodology followed to compare and assess the derived ARFs to a selection of ARF methods currently used in South Africa and internationally is discussed in the next section.

4.6 Extrapolation, Assessment and Comparison of ARFs

The applicable limits and ranges of the calibrated ARF algorithms for extrapolation to ungauged catchments were defined in terms of a range of different *A*, *D*, *T* and MAP values. The derived ARF algorithms were compared to a selection of other ARF estimation methods currently recommended for general use in South Africa, for example, by Van Wyk (1965), Wiederhold (1969) and Alexander (1980; 2001). It was also considered appropriate to compare the derived ARF algorithms with the UK FSR method (NERC, 1975) since a similar methodology as used in this study was implemented. The estimated relationship between ARFs and AEP was also reviewed with respect to the findings of these studies. Two different approaches were followed to compare and assess the derived ARFs to a selection of ARF methods as summarised below:

(a) **Approach 1 (Standard input variables):** The derived ARF algorithms were compared to the ARF estimation methods proposed by Van Wyk (1965), Wiederhold (1969) and Alexander (1980; 2001) by using standard input variables. The standard input variables for catchment areas (10 km^2 to 30 000 km^2), storm duration (1 hour to 72 hours) and rainfall intensity (50 to 200 $mm.h^{-1}$) as depicted in Figures 2.4 and 2.5 in Chapter 2, were used as inputs for Eqs. (2.5) to (2.10) in order to assess the consistency between the numerically and graphically derived ARF values. In each case, the ARF diagrams were reproduced in Microsoft Excel by manually extracting values from the original ARF diagrams (refer to Chapter 2, Figures 2.4 to 2.8). Thereafter, the graphical results, as obtained from each reproduced ARF diagram, were compared to the ARF values computed using Eqs. (2.5) to (2.10) in order to highlight any biases and inconsistencies present.

(b) **Approach 2 (Catchment level):** All the ARF estimation methods evaluated in (a) were compared and evaluated in 12 gauged catchments located in the C5 secondary drainage region in order to establish the biasness, consistencies or inconsistencies determined during the first approach as well as to establish the need for further research in this field. These 12 gauged catchments were selected since all the required catchment geomorphological variables (such as the catchment area), time parameters (such as the time of concentration or critical storm duration) and climatological variables (such as the design rainfall depths and intensities) were available and obtained from Gericke and Du Plessis (2011).

An Areal Reduction Factor Tool (ARFT) was developed in Microsoft Excel 2007 to automate the methodological procedures, as described above in Sections 4.1 to 4.6. Typically, the following worksheets are available in the ARFT: (i) general catchment information, (ii) observed daily rainfall data processing, (iii) areal and point AMS estimation, (iv) probabilistic plotting and analyses of point and areal AMS, (vi) sample ARF value estimation and (vii) multiple regression analyses to derive empirical ARF algorithms.

The use of the ARFT not only reduced the repetitive processing time of rainfall data analysis and sample ARF value estimations but also ensured that an objective and consistent approach was implemented.

The next chapter provides the results derived according to the methodology discussed in this chapter and an interpretation of the results.

CHAPTER 5 : RESULTS AND DISCUSSION

This chapter presents the research results obtained according to the methodology described in Chapter 4 with a discussion and critical assessment of these results.

5.1 Analysis of Rainfall Data

Table 5.1 indicates the number of rainfall stations used in each QC along with the minimum number of rainfall stations required based on the Siriwardena and Weinmann's (1996) criteria as discussed in Section 4.1, Chapter 4.

Table 5.1: Number of rainfall stations in relation to catchment size

QC	QC size (km²)	Number of stations used	Number of stations required
C51A	675	11	4
C51B	1 691	17	6
C51C	624	7	4
C51D	922	10	4
C51E	806	9	4
C51F	876	9	4
C51G	1 835	24	6
C51H	1 781	18	6
C51J	1 051	11	5
C51K	3 628	32	10
C51L	2 029	11	7
C51M	1 518	7	6
C52A	937	12	4
C52B	949	10	4
C52C	600	8	4
C52D	471	6	3
C52E	897	10	4
C52F	688	16	4
C52G	1 789	15	6
C52H	2 373	14	7
C52J	1 922	15	6
C52K	4 331	32	11
C52L	2 404	24	7

It is evident from Table 5.1 that the number of rainfall stations used exceeds the minimum number of rainfall stations required in each QC. On average, the number of rainfall stations used exceeded the Siriwardena and Weinmann (1996) criteria by a ratio of 2.6, while the individual ratios varied between 1.2 and 4. These results confirm that the number of rainfall stations considered would not have a negative impact when point rainfall depths are averaged over an area.

However, the accuracy of areal rainfall estimates is not only dependent on the number of rainfall stations, but is even more reliant on the actual quality of the rainfall data. Therefore, by considering the rainfall data quality extraction criteria, as discussed in Chapter 4 (*i.e.* a record length ≥ 30 years and geographical location in relation to a specific QC under consideration), the final set of rainfall stations was established.

5.1.1 Infilling of missing rainfall data

A relatively high percentage of infilled data was used to enable the derivation of the most representative geographically-centred and probabilistically correct ARFs. This is seen in Table 5.2 which lists the percentage of rainfall data infilling required for each QC.

Table 5.2: Percentage of rainfall data infilling in each QC

QC	Observed record length (years)	Infilled record length (years)	Total record length (years)	Percentage infilled (%)
C51A	552	141	693	20.3
C51B	775	347	1 122	30.9
C51C	349	120	469	25.6
C51D	506	194	700	27.7
C51E	410	130	540	24.1
C51F	389	151	540	28.0
C51G	1 040	424	1 464	29.0
C51H	741	429	1 170	36.7
C51J	496	208	704	29.5
C51K	1 623	681	2 304	29.6
C51L	556	236	792	29.8
C51M	410	94	504	18.7
C52A	582	186	768	24.2
C52B	460	240	700	34.3
C52C	362	118	480	24.6
C52D	289	71	360	19.7
C52E	558	142	700	20.3
C52F	777	439	1 216	36.1
C52G	788	202	990	20.4
C52H	711	157	868	18.1
C52J	667	293	960	30.5
C52K	1 559	585	2 144	27.3
C52L	1 191	465	1 656	28.1

Based on the results contained in Table 5.2, the observed rainfall data still represents 15 791 years, as opposed to the 6 053 infilled years, meaning that 72.3% of the total record lengths are based on observed data. At a QC level, the percentage of infilled data varies between 18.1% (C52H) and 36.7% (C51H). It is important to note that the infilling procedure was primarily used: (i) to infill missing data periods within the observation period, and (ii) to ensure that there is sufficient synchronisation and overlapping between the various rainfall station recordings within a particular group of stations at a QC level. However, in some cases, infilling was also used to extend the record length prior to the start of observations. In such cases, special care was exercised. An extension was only considered if a small percentage of the rainfall stations within a particular group is characterised by missing data prior to the start of the mutual data period which defines the majority of the rainfall stations within the group of stations under consideration. The rainfall stations with corresponding infilled record lengths are summarised in Table 5.3.

Table 5.3: Number of rainfall stations with corresponding infilled record lengths

QC	Number of rainfall stations	Data period (years)		Total record length (years)
		From	To	
C51A	11	1913/1914	1975/1976	63
C51B	17	1913/1914	1978/1979	66
C51C	7	1913/1914	1972/1973	60
C51D	10	1913/1914	1982/1983	70
C51E	9	1913/1914	1972/1973	60
C51F	9	1917/1918	1976/1977	60
C51G	24	1913/1914	1973/1974	61
C51H	18	1912/1913	1976/1977	65
C51J	11	1913/1914	1976/1977	64
C51K	32	1928/1929	1998/1999	71
C51L	11	1929/1930	1998/1999	70
C51M	7	1929/1930	1998/1999	70
C52A	12	1911/1912	1973/1974	63
C52B	10	1911/1912	1980/1981	70
C52C	8	1925/1926	1984/1985	60
C52D	6	1925/1926	1984/1985	60
C52E	10	1923/1924	1991/1992	69
C52F	16	1916/1917	1991/1992	76
C52G	15	1916/1917	1980/1981	65
C52H	14	1925/1926	1986/1987	62
C52J	15	1923/1924	1985/1985	63
C52K	32	1913/1914	1989/1990	77
C52L	24	1920/1921	1988/1989	69

It is evident from Table 5.3 that most of the SAWS rainfall stations have been operational since 1913. Typically, the infilling procedure ensured that the total record lengths exceeded 60 years for the QCs.

5.1.2 Conversion factors

The conversion factor procedure (Adamson, 1981) performed on the fixed interval AMS, as described in Chapter 4 (*cf.* Section 4.1.2), resulted in a continuous AMS of point and areal rainfall depths for durations of 1, 8, 16, 24, 72 and 168 hours. The conversion factors (Tables 2.1 and 2.2, Chapter 2) applied to the point and areal AMS remained constant for the various durations under consideration, *e.g.* 0.6 (D = 1-hour), 0.90 (D = 8-hour), 0.96 (D = 16-hour), 1.11 (D = 24-hour), 1.05 (D = 72-hour) and 1.02 (D = 168-hour).

5.1.3 Scaling factors

Table 5.4 provides a summary of the scaling factors applicable to the 23 QCs within the study area. The scaling factors are based on Eqs. (4.1) to (4.4), and the information contained in Tables 4.1 and 4.2, Chapter 4 (Smithers and Schulze, 2003). The scaling factors varied between: 0.54 and 0.62 (D = 1-hour), 0.90 and 1.01 (D = 8-hour), 1.03 and 1.13 (D = 16-hour), 1.18 and 1.24 (D = 24-hour), 1.35 and 1.38 (D = 72-hour), and 1.64 and 1.75 (D = 168-hour).

Table 5.4: Derived scaling factors in each QC for various durations

QC	Durations (hours)					
	1	**8**	**16**	**24**	**72**	**168**
C51A	0.549	0.902	1.040	1.180	1.379	1.709
C51B	0.549	0.902	1.039	1.180	1.379	1.710
C51C	0.548	0.901	1.038	1.180	1.377	1.709
C51D	0.545	0.899	1.035	1.180	1.381	1.728
C51E	0.547	0.901	1.037	1.180	1.375	1.708
C51F	0.550	0.903	1.040	1.180	1.368	1.685
C51G	0.548	0.902	1.038	1.180	1.379	1.713
C51H	0.547	0.901	1.037	1.180	1.375	1.705
C51J	0.549	0.902	1.039	1.180	1.364	1.673
C51K	0.603	0.986	1.113	1.227	1.353	1.643
C51L	0.612	1.012	1.131	1.240	1.359	1.660
C51M	0.622	0.991	1.129	1.234	1.353	1.642
C52A	0.542	0.898	1.032	1.180	1.383	1.740
C52B	0.541	0.896	1.030	1.180	1.384	1.747
C52C	0.540	0.896	1.030	1.180	1.384	1.751
C52D	0.541	0.897	1.031	1.180	1.384	1.746
C52E	0.541	0.897	1.031	1.180	1.384	1.747
C52F	0.540	0.896	1.030	1.180	1.384	1.749
C52G	0.546	0.902	1.036	1.181	1.383	1.741
C52H	0.542	0.900	1.035	1.181	1.383	1.745
C52J	0.544	0.899	1.034	1.180	1.382	1.733
C52K	0.581	0.957	1.084	1.210	1.372	1.696
C52L	0.610	1.007	1.127	1.238	1.361	1.665

The results listed in Table 5.4 are characterised by insignificant scaling factor variations in each QC for corresponding durations. The standard deviation results vary between 0.01 and 0.04. The estimated average scaling factors for each duration typically ranged from 0.56 (*D* = 1-hour), 0.92 (*D* = 8-hour), 1.05 (*D* = 16-hour), 1.19 (*D* = 24-hour), 1.38 (*D* = 72-hour) and 1.71 (*D* = 168-hour). In comparing the short duration (*D* ≤ 24-hour) conversion factors (Adamson, 1981) with the short duration average scaling factors (Smithers and Schulze, 2003), the similarities were quite evident. When the longer durations (*D* > 24-hour) were compared, however, some noticeable differences were evident. A comparison between the conversion and scaling factors as applicable to QC C51M is shown in Figure 5.1.

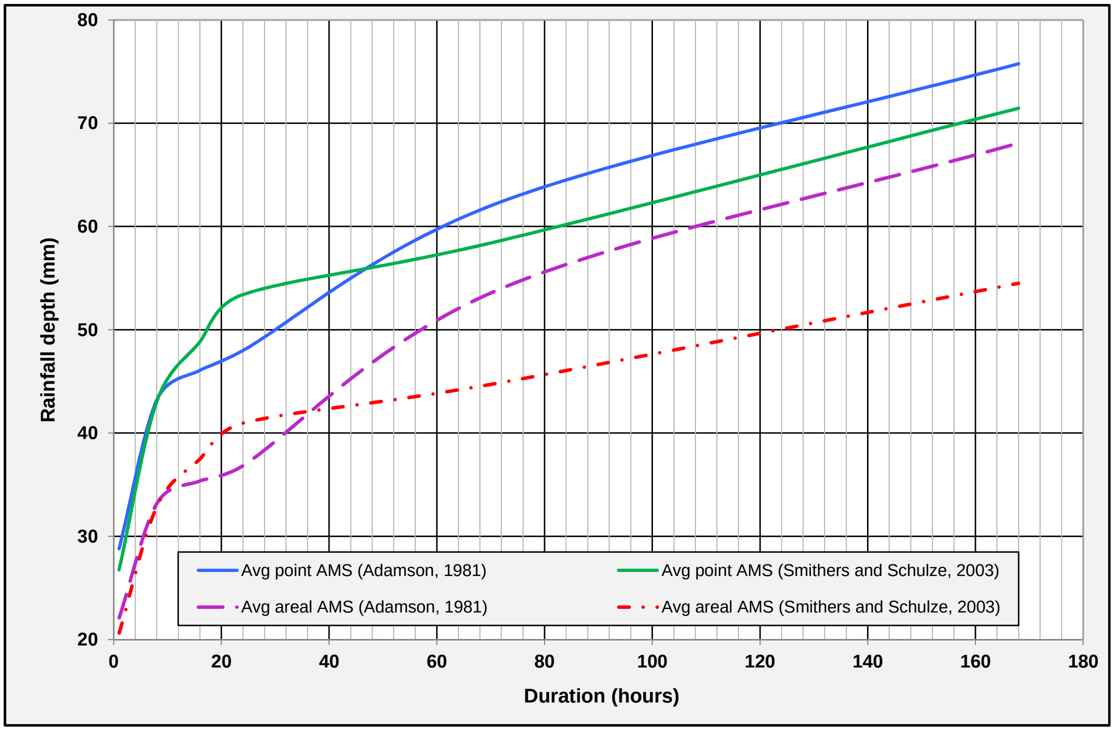

Figure 5.1: Comparison of conversion and scaling factors in QC C51M

The data presented in Figure 5.1 are based on a mutual data period of 70 years applicable to seven rainfall stations located in QC C51M. The average point and areal AMS plots shown are based on the fixed-interval (1-, 3-, and 7-day) AMS values initially extracted and converted or scaled as follows:

(a) **Conversion (Adamson, 1981):** The 1-day fixed interval AMS rainfall values were converted to a continuous rainfall (1-, 8- and 16-hour) series using the conversion factors as listed in Table 2.1, Chapter 2. The 3-day and 7-day fixed interval AMS values were converted to a continuous rainfall (24-, 72- and 168-hour) series using the conversion factors as listed in Table 2.2, Chapter 2.

(b) **Scaling (Smithers and Schulze, 2003):** The procedures as described in Section 4.1.3, Chapter 4 were followed to scale the 1-day fixed interval AMS to durations of 1-, 8-, 16-, 24-, 72- and 168-hour.

From Figure 5.1 it is evident that the scaling factors (Smithers and Schulze, 2003) tend to increase at a constant rate for durations > 24-hour. The latter tendency could be ascribed to the fact that the scaling factors were estimated from the mean of the 1-day AMS. The dissimilarities that exist between the conversion factors (Adamson, 1981) and scaling factors (Smithers and Schulze, 2003) for corresponding durations are conspicuous (*cf.* Tables 2.1, 2.2 and 5.4). Overall, the conversion factors (Adamson, 1981) resulted in higher rainfall depths for longer durations when compared to the scaling factors, since the actual long duration (3-day and 7-day) AMS data were used (*cf.* Section 4.1.2, Chapter 4). The Adamson methodology proved to be more robust with overall consistent results which are also endorsed by the wide diversity of observed data (1-day, 3-day and 7-day) being used. The Adamson methodology was therefore selected to derive the probabilistically correct ARFs in Section 5.4.

5.2 Averaging of Observed Rainfall

The Thiessen polygons generated from point feature classes (SAWS rainfall stations) for the C5 secondary drainage region are illustrated in Figure 5.2. Thiessen polygons were also generated for each individual QC to provide Thiessen weights for the estimation of each rainfall station's contribution to the daily point and areal rainfall in a particular QC. A summary of the SAWS rainfall stations and corresponding Thiessen weights at a QC level in the C51 and C52 tertiary drainage regions are listed in Tables A.4 and A.5, Appendix A, respectively.

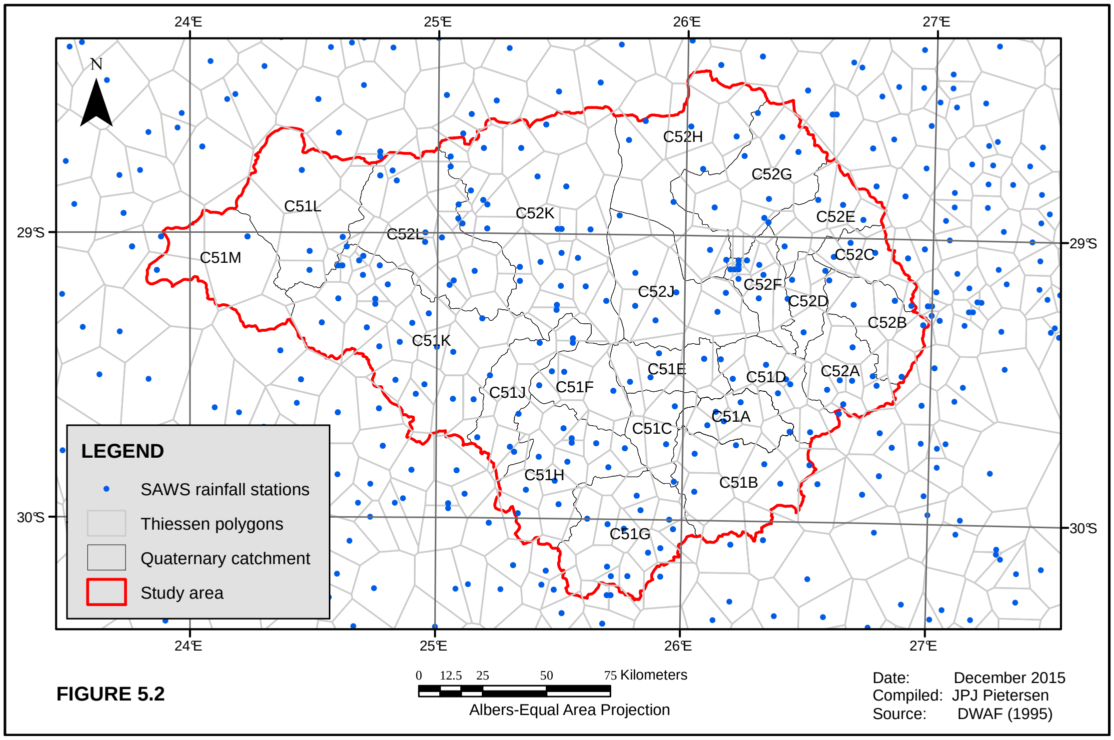

Figure 5.2: Layout of the Thiessen polygons in the C5 secondary drainage region

5.3 Probabilistic Analyses of Weighted AMS

An example of the probabilistic analyses results at 10-year ascending record length increments in QC C51M are listed in Table 5.5.

The areal design rainfall (based on the DCR approach; Section 2.5.3, Chapter 2) and design point rainfall (based on Approach 2; Section 4.3, Chapter 4) results are listed in Table 5.5. Generally, the results in Table 5.5 are characterised by a high variability, especially for record lengths ≤ 30 years. Both the point and areal design rainfall values are more consistent for longer record lengths. For instance, the variation between point and areal values for any RI is less significant with an increase in record length. Hence, a minimum infilled record length of 60 years was used for the probabilistic analyses, as highlighted in Chapter 4.

Table 5.5: Design point and areal design rainfall estimation results for various record lengths at C51M

	RI	Record length (years) for *D* = 24-hour						
		10	**20**	**30**	**40**	**50**	**60**	**70**
Areal design rainfall (mm)	**2**	35.4	32.0	35.3	37.1	35.9	34.4	33.6
	5	38.9	43.5	46.5	48.8	49.6	49.8	49.2
	10	39.6	49.4	51.8	55.0	58.1	60.9	59.9
	20	40.0	53.9	55.9	60.0	65.9	72.3	70.5
	50	40.1	58.5	59.8	65.3	75.4	88.2	84.6
	100	40.2	61.3	62.1	68.5	82.2	101.0	95.4
	200	40.2	63.6	63.9	71.2	88.6	114.7	106.5
Design point rainfall (mm)	**2**	47.5	45.0	48.7	50.2	49.3	47.2	46.0
	5	53.6	56.6	60.1	61.7	63.1	63.4	62.7
	10	55.3	61.9	65.0	67.0	70.7	73.9	72.6
	20	56.1	65.8	68.4	70.8	76.9	83.7	81.4
	50	56.6	69.5	71.4	74.4	83.8	96.1	91.8
	100	56.8	71.5	73.0	76.4	88.2	105.2	98.9
	200	56.9	73.0	74.1	77.9	92.0	114.0	105.5

Typical examples of the probabilistic plot results based on the ranked point and areal AMS values at QC C51M for various durations are respectively illustrated in Figures B.1 to B.6 and Figures B.7 to B.12, Appendix B.

In Figures B.1 to B.12, the LP3/MM probability distribution estimates are the lowest for the 2 and 5-year RIs, while being the second highest for the 100 and 200-year RIs. The GLO/LM estimates proved to be comparable with the results based on the GEV/MM and GEV/PWM probability distributions at RIs $\leq$ 5 years, however differences of up to 20% were evident at RI = 200-year.

Overall, the estimates based on the GEV/PWM probability distributions proved to be the most consistent in all the QCs under consideration. These results are also in agreement with the findings of Smithers and Schulze (2000a) and Siriwardena and Weinmann (1996) as highlighted in Section 4.3. Consequently, the GEV/PWM point and areal design rainfall values were used to derive sample ARFs in each QC under consideration.

5.4 Estimation of ARFs

The derived geographically-centred sample ARF results at a QC level in the C51 and C52 tertiary drainage regions (refer to Section 4.4 for the methodology) are listed in Tables A.6 and A.7 in Appendix A, respectively.

It is evident from Tables A.6 and A.7 that the sample ARF values derived are not constant and tend to increase with both an increase in RI and storm duration with sample ARF values that range from 0.581 (RI = 2-year) to 1.085 (RI = 200-year). The sample ARF values larger than unity (> 1) in some QCs are associated with RIs ≥ 100-year. In some cases, the sample ARF values deviated from the expected norm, for example, an increase in catchment area, with decreasing ARF values. Typically, larger ARF values are evident in some of the larger QCs as opposed to some of the smaller QCs. This phenomenon could also be associated with (i) the temporal and spatial variations that might exist within each QC and (ii) the presence of uniform rainfall covering the whole QC, irrespective of the catchment size under consideration.

The average sample ARF values for various durations and RIs for all 23 QCs in the C5 secondary drainage region were also estimated to highlight any consistencies and/or inconsistencies. At a secondary drainage region level, the average sample ARFs increased with an increase in duration and ranged from 0.78 (D ≤ 24-hour) to 0.91 (D = 72-hour) and 0.94 (D = 168-hour). Similarly, the average sample ARFs also increased with an increase in recurrence interval and the average ARF values range from 0.74 (RI = 2-year) to 0.90 (RI = 200-year). In most of the QCs under consideration, the ratio between point and areal design rainfall for various durations, namely, the sample ARFs, equalled unity (≈ 1) for RI ≥ 200-year.

Figures B.13 to B.18 in Appendix B illustrate the ratio between the point and areal design rainfall estimates used for the estimation of sample ARFs. Figures B.19 to B.41 in Appendix B show the variation of sample ARFs (within each QC) with the corresponding RIs. Apart from the possible presence of uniform rainfall events for the larger RIs, the areal average design rainfall depth or intensity is likely to be less than the maximum design point rainfall depths or intensities. This is confirmed by the results listed in Tables A.6 and A.7, Appendix A and also confirms the first study assumption, namely, that design point rainfall estimates are only representative for a limited area.

5.5 Derivation of ARF Algorithms

Backward stepwise multiple linear regression analyses with deletion generally resulted in the best prediction model for ARF at both a secondary and tertiary drainage region level. The following independent predictor variables were retained and included in the calibrated equation: (i) MAP (mm), (ii) *A* (km²), (iii) RI (years), and (iv) *D* (hours). At a confidence level of 95%, the above independent predictor variables contributed significantly towards the prediction accuracy and proved to be the best combination for estimating the ARF values.

However, the regression analyses at a secondary drainage level were characterised by a low degree of association between the observed (ARF_x) and estimated (ARF_y) values with the coefficient of determination (r^2) ≈ 0.3. Such low r^2 values are not only indicative of a low degree of association but also of the heterogeneity between ARFs in different QCs due to the non-uniform rainfall distribution. Therefore, by considering the latter, separate backward stepwise multiple linear regression analyses with deletion were conducted in each of the two tertiary drainage regions. Hence, the same equation format with different regional calibration coefficients (Table 5.6) was used in each of the two regions, namely, tertiary drainage regions C51 and C52. The derived ARF regression resulted in Eq. (5.1) shown below.

$$ARF_y = x_1 MAP + x_2 A + x_3 RI + x_4 D \tag{5.1}$$

Where:

ARF_y = estimated *ARF* (%),

A = catchment area (km^2),

D = duration (hours),

MAP = Mean Annul Precipitation (mm),

RI = recurrence interval (years), and

x_1 - x_4 = regional calibration coefficients (Table 5.6).

Table 5.6: Regional calibration coefficients applicable to Equation (5.1)

Region	Regional calibration coefficients (* 10^{-2})			
	x_1	x_2	x_3	x_4
C51	15.645	0.557	8.119	11.230
C52	13.784	0.333	5.783	11.831

Table 5.7: Summary of GOF statistics for the C51 and C52 tertiary drainage regions

ARF GOF (Eq. 5.1) results		
Criterion/Region	**C51**	**C52**
Confidence level [(1- α), %]	95	
Coefficient of multiple-correlation [Eq. (4.5)]	0.97	0.85
Standard error of estimate [Eq. (4.6), %]	9.78	6.55
F-Observed value (*F*-statistic)	9 006.0	18 604.4
Critical *F*-statistic (F_{α})	2.39	2.39

In considering the GOF statistics and hypothesis testing results, as listed in Table 5.7, it is evident that the best results are seen in the C52 tertiary drainage region, with the standard error of the ARF_y estimate = 6.6% and an associated coefficient of multiple-correlation = 0.85. Scatter plots of the ARF_y [Eq. (5.1)] and ARF_x values associated with all the QCs in each tertiary drainage region are shown in Figures 5.3 and 5.4 to highlight any regional differences.

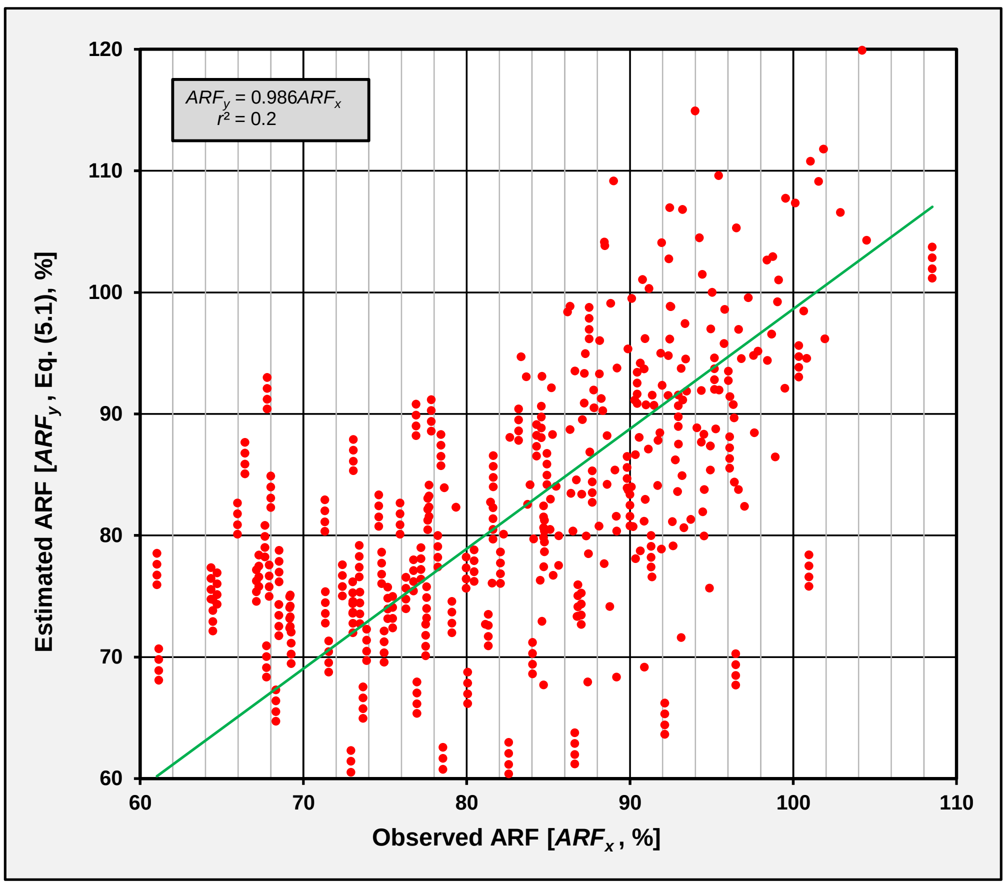

Figure 5.3: Scatter plot of the ARF_y [Eq. (5.1)] and observed ARF_x values of the C51 tertiary drainage region

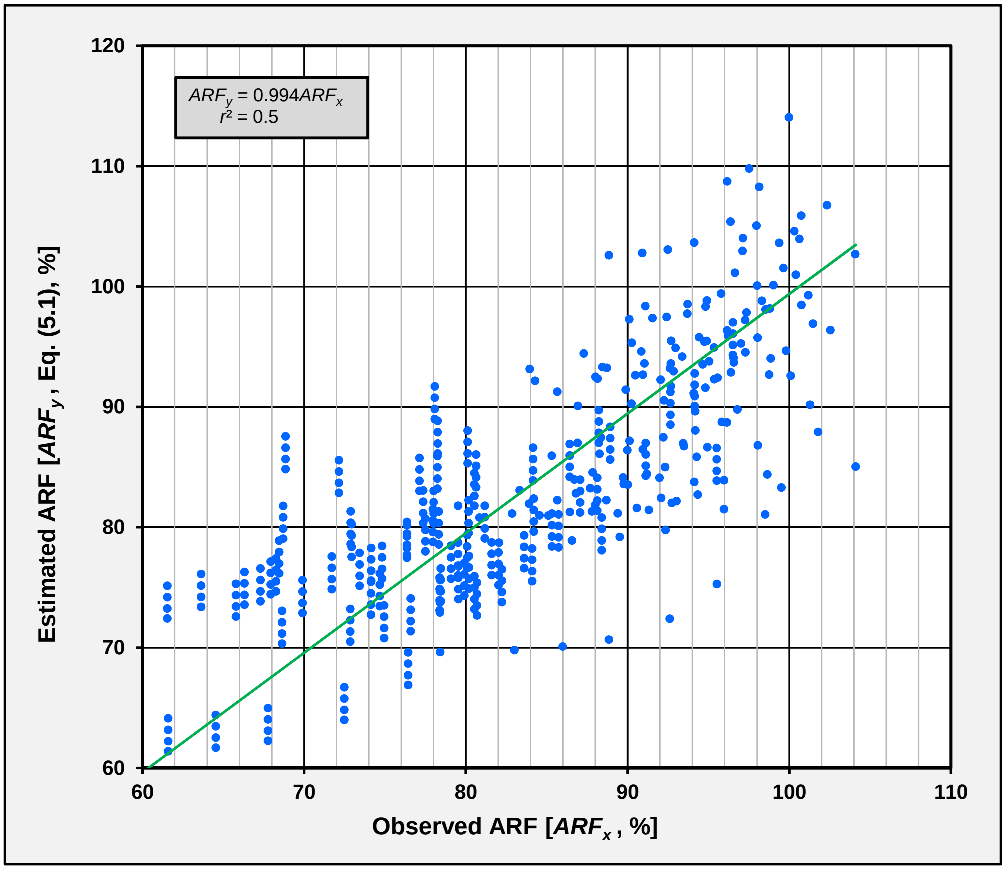

Figure 5.4: Scatter plot of the ARF_y [Eq. (5.1)] and observed ARF_x values of the C52 tertiary drainage region

In Figures 5.3 and 5.4, the ARF_y values computed using Eq. (5.1) showed a low to moderate degree of association with the observed ARF_x values, with r^2 values ranging between 0.2 (C51) and 0.5 (C52). Such low r^2 values are not only indicative of a low degree of association between the observed and estimated ARF values, but also highlight the heterogeneity between ARFs in different QCs due to the non-uniform rainfall distribution. The latter influence of non-uniform rainfall distribution on the ARF estimates was confirmed by the estimation of individual r^2 values at a QC level. Typically, the r^2 values between the ARF_x and ARF_y [Eq. (5.1)] values ranged from 0.48 ~ 0.79, thus confirming that rainfall distribution is more uniform over a smaller geographically-fixed area. Similarly, the individual r^2 values at a QC level in the C52 tertiary drainage region were considerably better than the r^2 value = 0.5, as depicted in Figure 5.4, with r^2 values ranging from 0.57 to 0.80. However, in the C51 tertiary drainage region, it is clearly evident that the ARF_y values computed using Eq. (5.1) are quite sensitive to MAP variability, especially for MAP values beyond the calibration range, *i.e.* values lower and higher than the minimum (326 mm) and maximum (576 mm) MAP values used during the calibration process. In addition, factors which might have an influence on the degree of association between ARF_y and ARF_x values in the different QCs are: (i) catchment shape, (ii) geographically-centredness of ARF estimates, (iii) nature of the non-uniform rainfall patterns not covering the whole catchment, and (iii) potential rainfall data quality discrepancies. Overall, the results discussed in this section also confirm the second study assumption, namely, that ARFs vary with predominant weather types, storm durations, climatological factors and recurrence intervals.

5.6 Comparison of ARF Estimation Methods

The results pertaining to the two different approaches (see Section 4.6, Chapter 4) followed to compare and assess the derived ARF_y values to a selection of ARF methods currently used in South Africa are summarised in this section. A default MAP value (520 mm) and corresponding RI (50-year) were used where applicable.

5.6.1 Approach 1: Standard input variables

In comparing the ARF estimation methods, it was noted, as expected, that some ARF estimates decreased with an increase in catchment area. Significant variations in the results also highlighted the presence of inconsistencies between the numerical and graphical ARF estimation methods. The comparison between Van Wyk's graphical (Figure 2.4) and numerical (Eq. 2.5) results, as shown in Figure 5.5, are characterised by increasing averaged percentage differences associated with an increase in the catchment area, for example 7.1% (10 km²), 7.8% (20 km²), 12.2% (50 km²), 18.3% (100 km²), 23.8% (200 km²), 27.3% (400 km²), and 28% (800 km²). A similar trend was also evident for the catchment area and rainfall intensities, in other words, an increase in rainfall intensity associated with a specific catchment area resulted in larger percentage differences. However, despite these percentage differences, an overall r^2 value of 0.96 confirmed the high degree of association between the ARFs estimated using the two methods. The comparison between Wiederhold's graphical approach (Figure 2.5) and Equations (2.7), (2.8a) and (5.1) is shown in Figure 5.6.

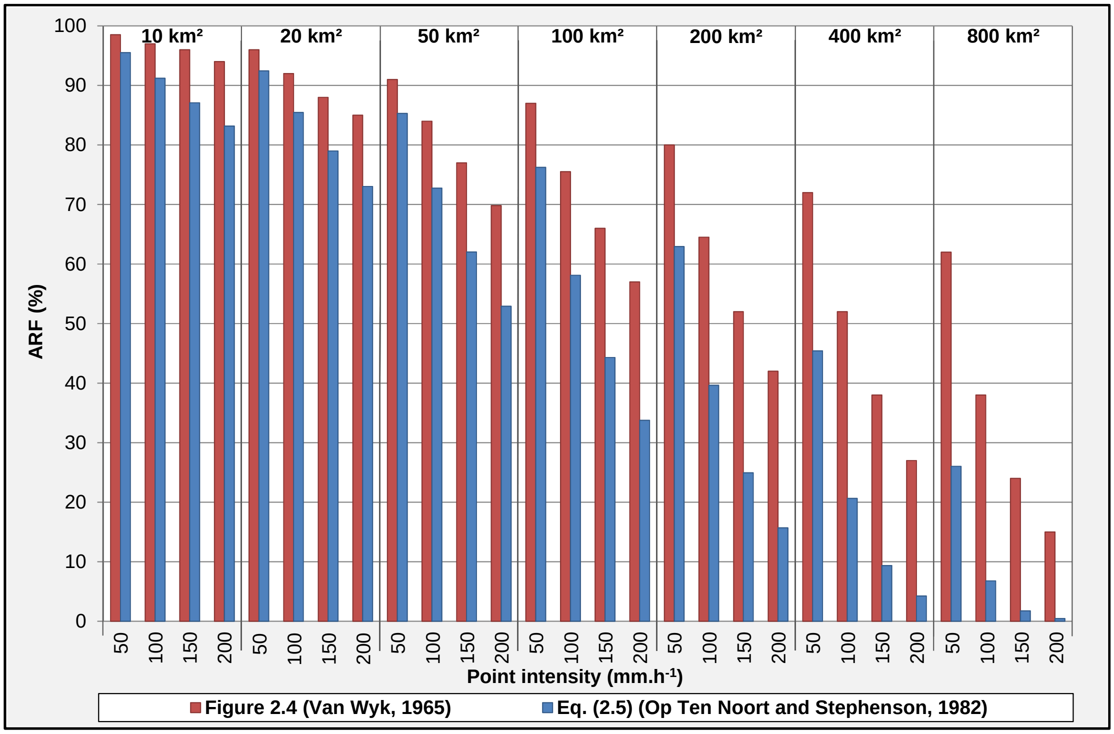

Figure 5.5: Comparison of the numerical vs. graphical storm-centred results (10 km² to 800 km²)

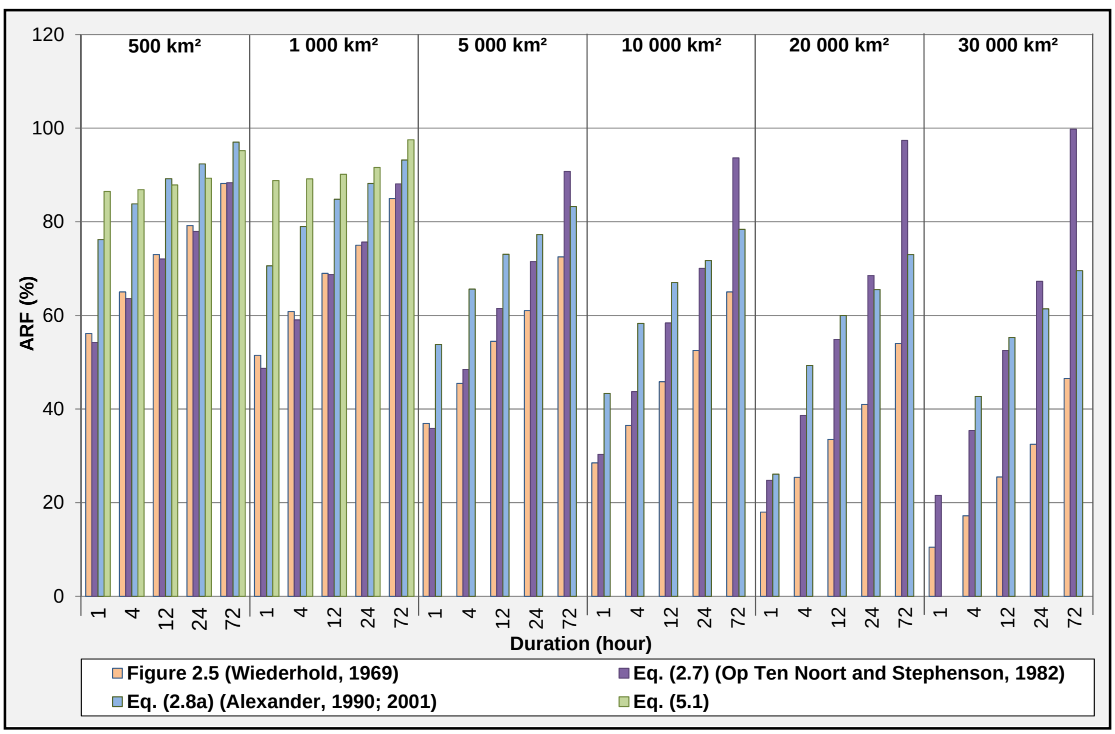

Figure 5.6: Comparison of the numerical vs. graphical geographically-centred results (500 km² to 30 000 km²)

The comparison between ARFs estimated using Wiederhold's graphical approach (Figure 2.5) and Eq. (2.7), as shown in Figure 5.6, are characterised by a high degree of association (r^2 = 0.92) and increasing percentage differences associated with an increase in the catchment area, for example averaged differences of 1.1% (500 km²), 1.7% (1 000 km²), 8.0% (5 000 km²), 13.6% (10 000 km²), 22.5% (20 000 km²) and 28.9% (30 000 km²). The ARF estimates also increased with increasing storm duration. It is important to note that, both Van Wyk's and Wiederhold's methods are storm-centred empirical methods which are not suitable for estimating catchment areal design rainfall from design point rainfalls. In doing so, the practising engineer would by default incorrectly assume that extreme design point rainfall and extreme areal design rainfall are produced by the same rainfall event or rainfall type. In Chapter 2 it was highlighted that, Alexander (1980) based his original methodology (Figure 2.7), on the UK FSR ARF diagrams (Figure 2.6, NERC, 1975), from which Op Ten Noort and Stephenson (1984), developed Eq. (2.8). In applying these approaches and Alexander's revised methodology [Figure 2.8 and Eq. (2.8a)], the results shown in Figures 5.7 and 5.8 were obtained.

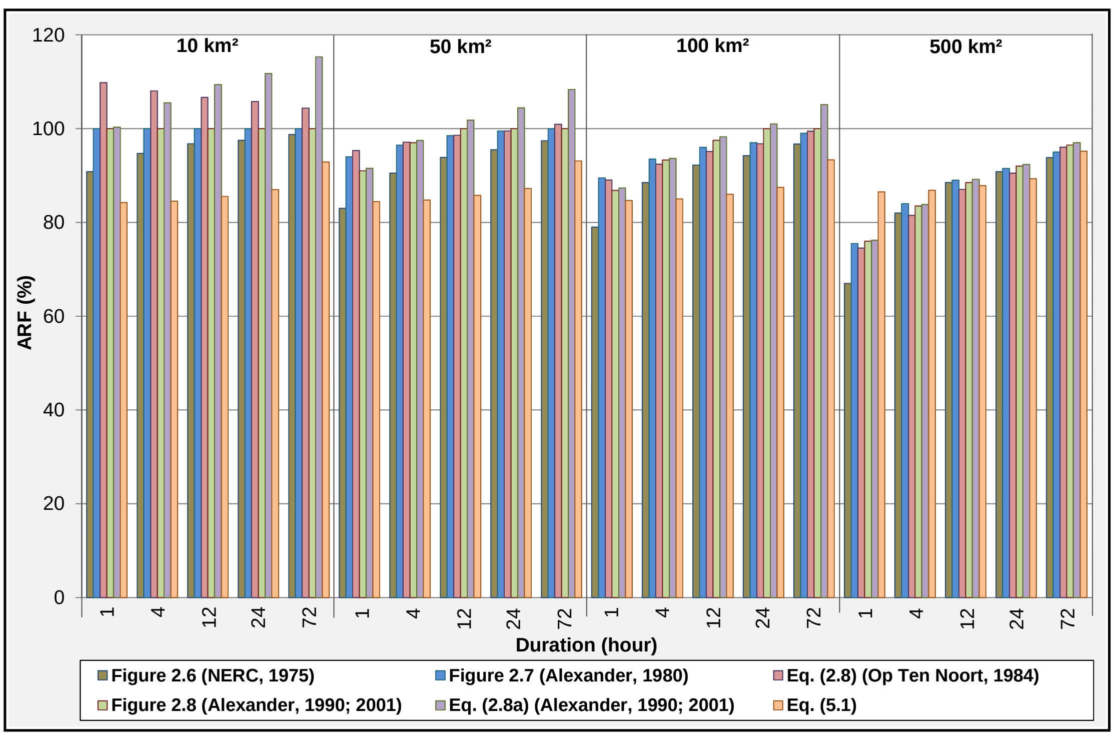

Figure 5.7: Comparison of the numerical vs. graphical geographically-centred results (10 km² to 500 km²)

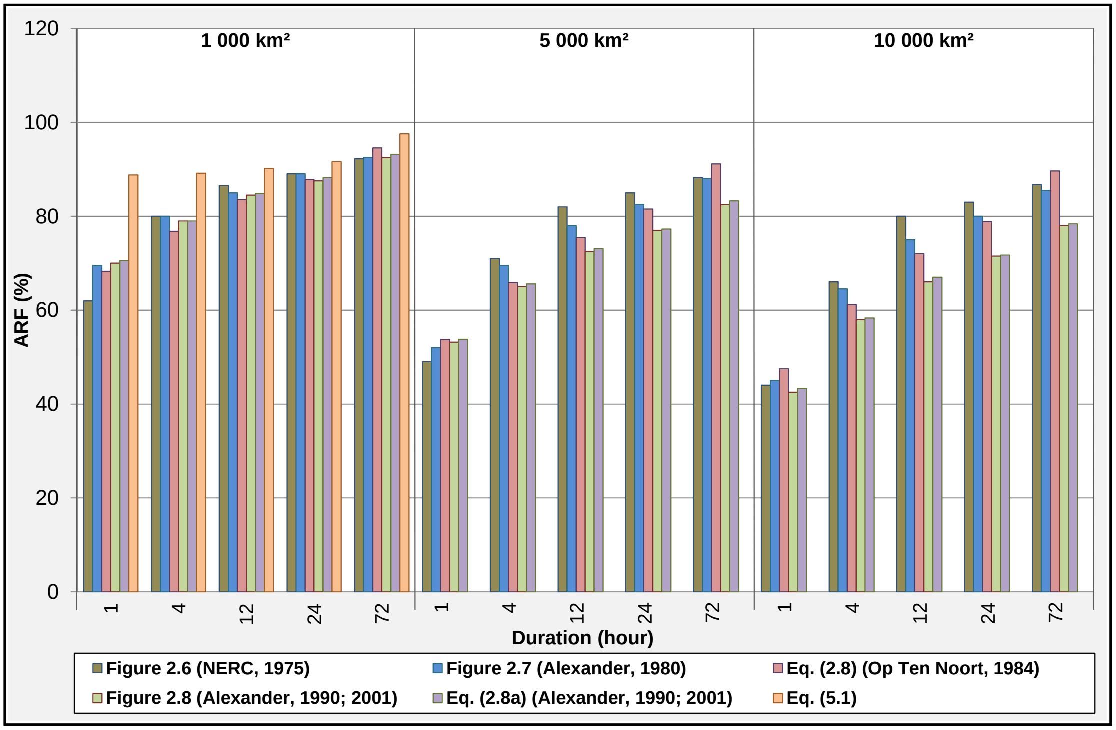

Figure 5.8: Comparison of the numerical vs. graphical geographically-centred results (1 000 km² to 10 000 km²)

In Figures 5.7, the larger percentage differences between the various methods were quite evident in smaller catchment areas (10 km^2 ≤ A ≤ 100 km^2), while Eq. (5.1) provided slightly lower ARF estimates compared to the other ARF methods for areas less than 500 km^2. However, Eq. (5.1) provided comparable ARF estimates for area ranges between 500 km^2 and 1 000 km^2 as depicted in Figures 5.7 and 5.8. The latter tendency also confirmed the findings of Alexander (1980), with specific reference to the occurrence of severe storm mechanisms that produce very high intensity rainfall with cell core areas exceeding the areal range and storm duration under consideration.

Typical average percentage differences between the various graphical and numerical ARF estimation methods and Eq. (5.1) as shown in Figures 5.7 and 5.8 are listed in Table 5.8.

Table 5.8: Summary of the average percentage differences between ARF estimates as illustrated in Figures 5.7 and 5.8

ARF estimation methods	Percentage differences
Eq. (5.1) vs. Eq. (2.8a)	1.4% ~ 21.6%
Figure 2.6 vs. Figure 2.7	2.2% ~ 5.6%
Figure 2.7 vs. Eq. (2.8)	0.2% ~ 6.9%
Figure 2.8 vs. Eq. (2.8a)	0.4% ~ 8.5%
Figure 2.5 vs. Eq. (2.8a)	15.4% ~ 21.4%

The r^2 values associated with the various methods listed in Table 5.8 are as follows: 0.41 [Eq. (5.1) vs. Eq. (2.8a); 0.91 (Figure 2.6 vs. Figure 2.7); 0.94 [Figure 2.7 vs. Eq. (2.8)] 0.96 [Figure 2.8 vs. Eq. (2.8a)]; and 0.87 [Figure 2.5 vs. Eq. (2.8a)]. Based on these results, it is evident that the geographically-centred numerical methods are generally more consistent and this could likely also be one of the reasons why these methods are preferred to the storm-centred approaches, especially if multi-centred storms are to be considered.

5.6.2 Approach 2: Catchment level

The application of the ARF estimation methods at a catchment level show some significant biases and systematic inconsistencies. These are summarised in Table A.8, Appendix A.

The results contained in Table A.8, Appendix A are characterised by percentage differences in estimated ARF values ranging from 17.6% to 27.6% in the smaller catchments (38 km² to 937 km²), 27.1% to 38% in medium-sized catchments (1 650 km² to 6 331 km²) and 44% to 71.5% in the large catchments (10 260 km² to 33 277 km²). Similar to the results shown in Figures 5.7 and 5.8, these comparisons showed that the geographically-centred numerical ARF estimation methods are more consistent. However, the geographically-centred ARF estimates did not account for the variation of ARFs with recurrence interval.

The results obtained from the comparison (Table A.8, Appendix A) were overall satisfactory for smaller catchment sizes (38 km^2 ~ 2 366 km^2) irrespective of the fact that the estimated design ARF_y values [Eq. (5.1)] were derived at a tertiary drainage region level. Equation (5.1) resulted in unrealistic ARF_y estimates for catchment areas exceeding 2 366 km^2. Typically, the ARF_y values computed using Eq. (5.1) increased with an increase in catchment size. The latter increase could be ascribed to the fact that some of the larger QCs had higher sample ARF values for all corresponding RIs. The latter findings also confirmed the third study assumption, namely, that the current South African ARF estimation methods are only applicable to specific temporal and spatial scales.

The final conclusions and recommendations for future research are discussed in the next chapter.

CHAPTER 6 : CONCLUSIONS AND RECOMMENDATIONS

6.1 Study Conclusions

This chapter contains a synthesised discussion of the research results presented in the previous chapter, which were gathered according to the methodology described in Chapter 4 with reference to the literature review in Chapter 2. This chapter also includes the conclusions and recommendations for future research based on the results obtained in Chapter 5.

6.1.1 Study objectives

The overall purpose of this study was to develop an enhanced methodology to express the spatial and temporal rainfall variability at a QC level by means of probabilistically correct ARFs.

The primary aim of this study was to estimate geographically-centred ARFs representative of the different rainfall producing mechanisms at a QC level in the C5 secondary drainage region. The focus was on the development of probabilistically correct ARFs, in other words, the relationship between T-year areal rainfall estimates and weighted average T-year point rainfall estimates for various A, MAP, D and RI values at a QC level were established.

The specific objectives identified to achieve the overall objective of this study are discussed in the subsequent sections.

6.1.2 Analyses of rainfall data

The number of rainfall stations available in the C5 secondary drainage region exceeds the minimum number of rainfall stations required per 500 km² (Siriwardena and Weinmann, 1996). However, the accuracy of areal rainfall estimates is not only dependent on the number of rainfall stations. The actual record length and quality of data are even more important. The SAWS daily fixed-interval rainfall data currently available for the C5 secondary drainage region are characterised by many incomplete recording lengths (≤ 60 years). The DREU (Lynch, 2004) was successfully used for the extraction and infilling of all the missing daily rainfall data series. Infilling is regarded as the last option since infilled values are based on the assumption of uniform temporal and spatial rainfall distribution between two distant rainfall stations.

Conversion (Adamson, 1981) and scaling (Smithers and Schulze, 2003) factors were considered and used to convert the daily point and areal rainfall series recorded at fixed 24-hour intervals to a continuous 24-hour rainfall series. The Adamson (1981) methodology proved to be more robust with more consistent results and was therefore used to derive probabilistically correct ARFs. In estimating short duration (≤ 24-hour) continuous *n*-hour rainfall series from fixed 24-hour rainfall, the Adamson (1981) conversion and Smithers-Schulze (2003) scaling factors resulted in comparable results. However, the Adamson conversion factors could be regarded as being out-dated (1981) and only one set of conversion factors are available for the whole of South Africa. In contrast, the Smithers-Schulze (2003) scaling factors are a unique set of different regional scaling factors used to downscale the mean 1-day AMS values. In estimating long duration (> 1 day) continuous *n*-day rainfall series from fixed 24-hour rainfall, the Adamson (1981) methodology, which utilises accumulated daily rainfall totals and an '*n*-day sliding window' approach, is preferred to the upscaling of mean 1-day AMS values as proposed by Smithers and Schulze (2003). The downscaling (used in both the Adamson and Smithers-Schulze methodologies) and upscaling (used only in the Smithers-Schulze methodology) of 1-day AMS values to short and long durations respectively resulted in sample ARF estimates with little or no variation between storm durations.

Based on the above, it is evident that actual *n*-hour (≤ 24-hour) or *n*-day (> 1-day) AMS rainfall series are required to estimate representative sample ARF values. As highlighted in Chapter 2, long duration rainfall monitoring (≥ 1-day) could be regarded as sufficient in South Africa. However, short duration rainfall monitoring (≤ 24-hour) is only limited to 412 sub-daily rainfall stations with a low reliability due to several errors such as missing data and differences (> 20 mm) between the digitised and standard rain gauge daily totals.

The primary aim of this study was to estimate geographically-centred ARFs representative of the different rainfall-producing mechanisms at a QC level. Sample ARFs therefore need to be specific to the QC and could not be extrapolated to other QCs with a markedly different shape, for instance circular vs. elongated. The differences in catchment shape and in the rainfall distribution patterns, storm direction and movement, whether aligned along the catchment or perpendicular to it, mean that different ARF estimates could be expected. The latter results are also in agreement with the findings of Veneziano and Langousis (2005).

6.1.3 Averaging of observed rainfall

Based on the findings from this study, as well as the preferential use of the Thiessen polygon method in various international ARF studies, such as those by Bell (1976), Stewart (1989) and Siriwardena and Weinmann (1996), the Thiessen polygon method is recommended for future use in ARF estimation studies. However, the Isohyetal method would be the preferred method to determine average areal design rainfall depths in catchments where rainfall stations have a poor areal distribution and the catchment topography is highly variable.

6.1.4 Probabilistic rainfall analyses

Overall, the estimates based on the GEV/PWM probability distributions proved to be the most consistent in all the QCs under consideration. These findings are in agreement with the findings of Siriwardena and Weinmann (1996). The DCR approach (Van der Spuy and Rademeyer, 2014) used to estimate areal design rainfall shows promising results. The DCR approach further highlights that ARFs are not necessarily required when Thiessen weights are applied to daily observed point rainfall values for the purpose of estimating catchment rainfall.

6.1.5 Estimation and comparison of ARFs

The derived geographically-centred sample ARF values applicable to the C5 secondary drainage region are not constant and tend to increase with both an increase in RI and storm duration. Assumption 1 mentioned in Section 1.3.2 states that design point rainfall estimates are only representative for a limited area and for larger areas. This has been confirmed because the areal average design rainfall depth or intensity is likely to be less than the maximum design point rainfall depths or intensities, apart from the possible presence of uniform rainfall events for the larger RIs.

The differences evident between the estimated sample ARFs in the C51 and C52 tertiary drainage regions emphasised the need for regionalisation. Typically, the ARF diagram (Alexander, 2001), available in the Drainage Manual (SANRAL, 2013) and recommended for general use in South Africa, does not take into account any regional differences while also not being probabilistically correct. For example, the relationships between *T*-year areal rainfall and weighted average *T*-year point rainfall estimates are not recognised.

6.1.6 Achievement of objectives and major findings

An enhanced methodology to express the spatial and temporal rainfall variability at a QC level by means of probabilistically correct ARFs was developed. The geographically-centred ARFs are representative of the different rainfall-producing mechanisms at a QC level in the C5 secondary drainage region. Further to this, they are regarded as being probabilistically correct seeing that the relationships between *T*-year areal rainfall estimates and weighted average *T*-year point rainfall estimates for various *A*, MAP, *D* and RI values at a QC level were established.

The major findings of the study are as follows:

(a) Design point rainfall estimates are only representative of a limited area and for larger areas the areal average design rainfall depth or intensity is likely to be less than the maximum design point rainfall depths or intensities.

(b) ARFs vary according to predominant weather types, storm durations, climatological factors and recurrence intervals.

(c) The use of a geographically-centred approach based on a modified version of Bell's (1976) method has proved to be appropriate for the study undertaken bounded within a 'fixed' catchment area, namely, at a QC level.

(d) The derived ARF algorithm(s) [Eq. (5.1)] provided improved estimates when compared to the geographically- and storm-centred ARF methods currently used in South Africa. Typically, the geographically-centred methods presently used in South Africa were either transposed from methods developed in the UK with little local verification or were developed using very limited local data. Furthermore, storm-centred ARF methods (Van Wyk, 1965; Wiederhold, 1969) are wrongfully applied in a geographically-centred manner.

6.2 Recommendations for Future Research

In view of the improved results obtained from this study, the ARF_y regressions to estimate the ARF_x values should be expanded to other catchments in South Africa by taking cognisance of the recommendations for future research stated below:

(a) **Regionalisation:** A regionalisation scheme (Hosking and Wallis, 1997) for ARF estimation in South Africa should be adopted or developed.

(b) **Estimation of index ARF values at ungauged sites:** Once the method of regionalisation has been selected, the procedures to apply the method at ungauged sites need to be developed. This will require the estimation of scaling parameters (*e.g.* index ARF parameters) at ungauged sites as a function of site characteristics or the development of a means to transfer the hydrological information from gauged to ungauged sites within a region.

(c) **Circular test catchments:** Since different ARF estimates could be expected due to differences in the catchment shape and size, the use of QCs (with a unique shape, orientation and size) to derive sample ARFs limits the potential of extrapolation beyond the QC boundaries. Therefore, the use of multiple circular catchments with random sizes (*e.g.* 125, 250, 500, 1 000, 2 000, 4 000 and 8 000 km^2) covering a specific region will enable the estimation of ARFs over a specific region.

(d) **Development of a software interface:** An interface to enable practitioners to apply and use the regionalised ARF equations should be developed. The software should allow for the implementation of the proposed methodology on a national scale in South Africa.

6.3 Conclusion

The development of an improved methodology for expressing spatial and temporal rainfall variability at a QC level through the use of probabilistically correct ARFs was the objective of this research. The achievement of this objective and the sub-objectives, as based on the research results, as well as the recommendations stemming from this have been discussed in this chapter. It is envisaged that the implementation of both the identified research values and recommendations for future research will ultimately contribute fundamentally to both improved ARF and peak discharge estimations in South Africa.

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APPENDIX A: TABULATED INFORMATION AND RESULTS

Table A.1: Summary of empirical ARF estimation methods used internationally

Approach	Method	Mathematical algorithm	Origin	Comments
Geographically-centred	**USWB method** (USWB, 1957; 1958)	$$ARF = \frac{N\sum_{i=1}^{N}\sum_{j=1}^{n} w_i \overline{\overline{P_{ij}}}}{\sum_{i=1}^{N}\sum_{j=1}^{n} P_{ij}} \quad \text{(A1)}$$ Where: ARF = areal reduction factor, N = number of stations within the catchment area, n = record length (years), $\overline{\overline{P_{ij}}}$ = point rainfall for station i on the day of the annual maximum areal rainfall in year j (mm), P_{ij} = annual maximum point rainfall of station i in year j (mm), and w_i = Thiessen weighted factor for station i.	USA	• Observed rainfall records (10 to 15 years of data) from dense rainfall monitoring networks in catchment areas (250 km² to 1 000 km²) were used. • Rainfall record lengths were regarded as insufficient to establish the effect of recurrence interval/AEP on the point-area rainfall relationships. • The areal rainfall of each event and associated duration was estimated using Thiessen weights. • The mean of the AMS was estimated, while the highest point rainfall measurement at each station in a particular year was selected.
Geographically-centred	**UK FSR method** (NERC, 1975)	$$ARF = \frac{1}{nN}\sum_{i=1}^{N}\sum_{j=1}^{n}\left(\frac{\overline{\overline{P_{ij}}}}{P_{ij}}\right) \quad \text{(A2)}$$ Where: ARF = areal reduction factor, N = number of stations within the catchment area, n = record length (years), $\overline{\overline{P_{ij}}}$ = point rainfall for station i on the day of the annual maximum areal rainfall in year j (mm), and P_{ij} = annual maximum point rainfall of station i in year j (mm).	UK	• Thirteen catchment areas (10 km^2 to 18 000 km²) and storm durations ranging from 2 minutes to 25 days were used. • Nation-wide UK rainfall records were used for the development of an ARF estimation diagram with catchment area and storm duration as variables. • ARF values were assumed to fit an average recurrence interval of between 2 to 3 years; however, recurrence interval/AEP was not taken into account, since the effect thereof was regarded as insignificant.

Table A.1: (continued)

Approach	Method	Mathematical algorithm	Origin	Comments
Geographically-centred	**Bell's method** (Bell, 1976)	$ARF_m = \dfrac{\sum_{i=1}^{N}\left(w_i \overline{\overline{P_{ij}}}\right)_m}{\sum_{i=1}^{N}\left(w_i P_{ij}\right)_m}$ (A3) Where: ARF_m = areal reduction factor (ratio of areal rainfall of rank *m* to the Thiessen weighted average point rainfall of the same rank (%), m = rank value, N = number of stations within the catchment area, $\overline{\overline{P_{ij}}}$ = point rainfall for station i on the day of the annual maximum areal rainfall in year *j* (mm), P_{ij} = annual maximum point rainfall of station *i* in year *j* (mm), and w_i = ratio of the areal rainfall of rank *m* to the Thiessen weighted average point rainfall of the same rank.	UK	• Based on the derivation of frequency curves of areal and average point rainfall. • Estimate ARFs from the ratio of areal to average point rainfall at the relevant AEPs. • Areal rainfall is determined from Thiessen weights of the annual maximum point rainfall values. • More probabilistically correct ARFs compared to the USWB, NERC and the Desbordes et al. (1984) methods. • Dependant on the recurrence interval • Significantly lower ARFs for high AEPs (20–100 years) were obtained. • This method showed a tendency towards lower ARFs with longer AEPs for shorter duration (24 hour and less) rainfall events.
Geographically-centred	**Stewart's method** (Stewart, 1989)	$ARF_T = \dfrac{P_{AS(T)}\overline{P_A}}{P_{PS(T)}\overline{P_P}}$ (A4) Where: ARF_T = areal reduction factor at a specific AEP (%), $\overline{P_A}$ = mean of annual maximum areal rainfall (mm), $P_{AS(T)}$ = standardised *T*-year areal rainfall (mm), $\overline{P_P}$ = mean of annual maximum point rainfall (mm), and $P_{PS(T)}$ = standardised *T*-year point rainfall (mm).	UK	• Based on Bell's method (1976) using daily rainfall data from north-west England. • A total of 834 rainfall stations with at least 25 years of data were used • A total of 544 sample catchments (25 km² to 10 000 km²) and storm durations ranging from 1 day to 8 days were analysed. • ARFs were expressed as a function of the geographical location and AEP • ARFs decreased with an increasing catchment area and AEP. • ARF estimates proved to be significantly lower than those based on the UK FSR method (NERC 1975).

Table A.1: (continued)

Approach	Method	Mathematical algorithm	Origin	Comments
Geographically-centred	**Omolayo's method** (Omolayo, 1993)	*ARF* = $\dfrac{P_A(A,T_d,T)}{\dfrac{1}{\sum_i w_i}\sum_i\left[w_i\overline{P_P}(T_d,T)\right]}$ (A5) Where: *ARF* = areal reduction factor, *A* = catchment area under consideration (km²), *n* = record length (years), P_A = *T*-year areal rainfall (mm), $\overline{P_P}$ = average *T*-year point rainfall (mm), T_d = storm duration (hours), and w_i = weighted average P_P of the gauges *i* in the same region.	Australia	• Daily rainfall data (30 years record length) were used. • The 1-day ARFs for the USA were transposed to Australia, given that the climatological variables were similar. • Probabilistically correct ARF estimation. • ARFs are defined as the ratio between areal rainfall and point rainfall of the same recurrence interval/AEP.
Geographically-centred	**Modified Bell's method** (Siriwardena and Weinmann, 1996)	*ARF* = $1-0.4\left(A^{0.14}-0.7\log T_d\right)T_d^{\,0.48}+0.002A^{0.4}T_d^{\,0.41}\left[0.3+\log\left(\dfrac{1}{T}\right)\right]$ (A6) Where: *ARF* = areal reduction factor, *A* = catchment area (km²), *T* = recurrence interval (years), and T_d = storm duration (hours). **Ranges of application:** 1 km² ≤ *A* ≤ 10 000 km² 0.05 ≤ *AEP* ≤ 0.0005 18 hours ≤ T_d ≤ 120 hours	Australia	• AMS of areal and point rainfall were used instead of the PDS curtailed to a common base period as originally proposed by Bell (1976). • Over 2 000 daily rainfall stations in Victoria, Australia were used. • ARF values were estimated for a number of 'circular sample catchment areas' distributed through areas characterised by a high density rainfall-monitoring network. • ARF values were estimated for rainfall durations (1 to 3 days), catchment areas (125 to 8 000 km²) and recurrence intervals (2 to 200 years).
Geographically-centred	**Mixed gamma distribution method** (Yoo *et al.*, 2007)	$ARF_{(A,\,T)} = 1-Me^{-\left(aA^{b}\right)^{-1}}$ (A7) Where: *ARF* = areal reduction factor, *A* = rainfall storm areas (km²), *M, a, b* = parameters associated with each recurrence interval, and *T* = recurrence interval (years).	Korea	• A total of 25 rainfall stations with at least 30 years of record were used in a catchment area of 9 843 km². • Method utilises daily rainfall data instead of probabilistic curve fitting of the AMS. • Recurrence intervals ranging from 2 to 1 000 years were considered.

Table A.1: (continued)

Approach	Method	Mathematical algorithm	Origin	Comments
Storm-centred	**Annual maxima-centred method** (Asquith and Famiglietti, 2000)	$ARF = \frac{\int_0^R 2rS_{T(r)}\Delta r}{R^2}$ (A8) Where: ARF = areal reduction factor, A = rainfall storm areas (km²), R = maximum radius of circular catchment or integration limit (km), r = radius of concentric circle within the catchment (km), and $S_{T(r)}$ = ratio between rainfall depth at a specific location, distance r from the point of the design storm and the annual maxima rainfall.		• Method developed for the Austin, Dallas, and Houston regions, USA with a dense rainfall-monitoring network. • The Austin region (15 600 km²) had 108 daily rainfall stations, Dallas region (21 000 km²) had 103 daily rainfall stations and Houston region (35 800 km²) had 193 daily rainfall stations. • Several record lengths exceeded 80 years. • Method focuses on the analysis of the areal rainfall distribution to estimate ARFs for design storms. • ARFs decrease rapidly with increasing AEPs.
Analytical-empirical	**National Weather Service method** (Myers and Zehr, 1980; cited by Svensson and Jones, 2010)	$ARF = \frac{\overline{P_A}(f,\Delta t,A)}{\overline{P_P}(f,\Delta t,0)}$ (A9) Where: ARF = areal reduction factor, $\overline{P_A}$ = average areal rainfall for a specific frequency (*f*), duration (Δt) and area (A) (mm), and $\overline{P_P}$ = point rainfall for a specific frequency (*f*), duration (Δt) and area (A) (mm).	USA	• Method is based on the probabilistic analysis of rainfall AMS pair values of individual stations and the distance between these stations. • Rainfall depth-area curves were developed from a dense rainfall-monitoring network. • Effect of recurrence interval/AEP on ARFs is included, *i.e.* probabilistically correct ARFs. • ARFs decrease with increasing recurrence intervals. • ARFs not regarded as representative of the spatial and temporal rainfall variability. • Very complex approach and difficult to implement in practice.

Table A.2: Summary of analytical ARF estimation methods used internationally

Approach	Method	Mathematical algorithm	Origin	Comments
Spatial correlation	**Rodriquez-Iturbe-Mejia method** (Rodriquez-Iturbe and Mejia, 1974; cited by Svensson and Jones, 2010)	$ARF = \sqrt{E(\rho(d))}$ (A10) Where: *ARF* = areal reduction factor, and *E (ρ(d))* = expected correlation coefficient for the characteristic correlation distance.	Various	• Simple ARF estimation approach used in various areas. • Based on a spatial correlation structure using either an exponentially decaying function or a Bessel-type correlation structure. • Dependent on all observed rainfall data, *i.e.* the primary data and not only the AMS. • 'Design storm' areal rainfall distributions are not included.
Storm movement	**Storm movement method** (Bengtsson and Niemczynowicz, 1986)	$ARF = \frac{L_P}{L} = \frac{vT_d}{L}$ if $L_P < 0.5$ $ARF = 1 - \frac{0.25L}{L_P} = 1 - \left(\frac{0.25L}{vT_d}\right)$ if $L_P \geq 0.5$ (A11) Where: *ARF* = areal reduction factor, L = catchment length (km), L_P = extension of block rain cell (km), T_d = storm duration (hours), and v = storm speed (m.s^{-1}).	Sweden	• Represents the relationship between rainfall movement and ARFs. • ARFs are based on the limited extension of rain cells, movement and spacing between rain cells and the effect of rain cells on each other. • ARFs were obtained from point rainfall hyetographs and storm speeds. • Relations were established between moving storm-derived ARFs and ARFs estimated by a dense rainfall-monitoring network. • ARFs proved to be constant in Norway.

Table A.2: (continued)

Approach	Method	Mathematical algorithm	Origin	Comments
Spatial correlation	**Omolayo's method** (Omolayo, 1989; cited by Svensson and Jones, 2010)	LN distributed rainfall: $ARF_1 = Exp\left\{K_T \sigma \left(\sqrt{\frac{1+(N-1)\rho}{N}} - 1\right)\right\}$ (A12a) Normal distributed rainfall: $ARF_2 = \sqrt{\frac{1+(N-1)\rho}{N}}$ (A12b) Normal distributed rainfall (large number of rainfall stations): $ARF_3 = \sqrt{\rho}$ (A12c) Where: ARF = areal reduction factor, K_T = frequency factor corresponding to recurrence interval, N = number of rainfall stations, T = recurrence interval (years), σ = standard deviation of rainfall depth in the log domain (mm), and ρ = average spatial correlation coefficient.	Australia and USA	• Based on the average spatial correlation and the number of rainfall stations within an area. • Rainfall depths are assumed to be log-normally distributed. • Recurrence interval is considered • The normal distribution expression is similar to the relationship derived by Rodriguez-Iturbe and Mejia (1974), except that the correlation coefficient is averaged over the rainfall stations. • ARFs vary directly with the spatial correlation coefficient and inversely with standard deviation, number of rainfall stations and AEPs.
Crossing properties	**Bacchi-Ranzi method** (Bacchi and Ranzi, 1996)	$ARF = \frac{T_{A,Td}(F')}{T_A(F')}$ (A13) Where: ARF = areal reduction factor, A = area under consideration (km²), F' = F-quantile of the corresponding probability distribution, T_d = duration within the space-time domain where the rainfall process can be assumed uniform (hours), and T = recurrence interval (years).	Italy	• Sixteen Constant Altitude Plan Position Indicator (CAPPI) maps were recorded and analysed from the C-band weather radar to be compared with the corresponding rainfall data from 17 rainfall stations. • Based on the analysis of the crossing properties of the spatial and temporal rainfall process. • High rainfall intensity processes were assumed to be Poisson distributed. • ARF expressed as the ratio of areal and point rainfall intensity values associated with the same duration and frequency. • ARFs are dependent on the recurrence interval and catchment area.

Table A.2: (continued)

Approach	Method	Mathematical algorithm	Origin	Comments
Spatial correlation	**Sivapalan-Blöschl method** (Sivapalan and Blöschl, 1998)	$$ARF\left[k^2\left(\frac{A}{\lambda^2}\right),T_d,T\right]=\frac{b(T_d)c(T_d)k^2F_2\left(k^{-2}\right)-\frac{k^2}{F_1\left(k^{-2}\right)}\ln\left[\ln\left(\frac{T}{T-1}\right)\right]}{b(T_d)c(T_d)-\ln\left[\ln\left(\frac{T}{T-1}\right)\right]}$$ (A14) Where: *ARF* = areal reduction factor, *A* = catchment area (km²), *b* = function of duration, where $b(T_d)=-0.05+0.25T_d^{0.49}$ *c* = function of duration, where $c(T_d)=0.2+20T_d^{-0.7}$ $F_1(k^{-2})$ = generic properties of the gamma distribution, $F_2(k^{-2})$ = generic properties of the gamma distribution, k^2 = rainfall correlation structure, *T* = recurrence interval (years), T_d = storm duration (hours), and λ = spatial correlation length (km).	Austria	• Based on a spatial correlation structure using both extreme value and/or parent distributions. • ARF values are dependent on the catchment area, storm duration (spatial correlation structure) and recurrence interval. • The ARF values are independent of the rainfall regime. • ARF values decrease with an increasing catchment area and recurrence interval. • Method is rather regarded as a 'geographically-centred' method as opposed to 'storm-centred'. • The final ARF expression is regarded as complex and not user-friendly.
Scaling relationship	**De Michéle's method** (De Michéle *et al.*, 2001; cited by Svensson and Jones, 2010)	$$ARF=\left[1+\omega\left(\frac{A^z}{T_d}\right)^b\right]^{-\frac{v}{b}}$$ (A15) Where: *ARF* = areal reduction factor, *A* = catchment area (excluding the rain gauge area) (km²), T_d = storm duration (hours), and *b, υ, ω, z* = fitted parameters.	Italy	• Only eight years of rainfall data were used. • Storm durations (20 minutes to 6 hours) and catchment areas (0.25 km² to 300 km²) were used. • Recurrence intervals or AEPs were not included. • Method proved to be most reliable for storm durations between 1 hour and 3 hours, while less satisfactory for 20 minute and 6 hour storm durations. • Kriging was used to estimate the rainfall intensity AMS.

Table A.2: (continued)

Approach	Method	Mathematical algorithm	Origin	Comments
Radar data	**Polar 55C method** (Lombardo *et al*., 2006)	$$ARF = \frac{i_A(T_d,T)}{i_{A=1}(T_d,T)} \quad \text{(A16)}$$ Where: *ARF* = areal reduction factor, *A* = area under consideration (km²), *i* = rainfall intensity (mm.h^{-1}), *T* = recurrence interval (years), and T_d = storm duration (hours).	Italy	• The ARF values were estimated by using radar reflectivity maps collected with Polar 55C. • Rainfall intensities over the radar scanning region were estimated for durations (1 to 120 minutes) and recurrence intervals (2 to 50 years) by using the Arithmetic mean and Thiessen polygon methods. • The radar rainfall estimates were integrated for heavy rainfall data over an area of 900 km². • The radar used in this study is located 15 km south-east of Rome. • Study focussed of the influences of area, storm duration, intensity and recurrence interval on ARF variation. • The ARFs exceeded unity in small areas characterised by relative longer storm durations.

Table A.3: Daily SAWS rainfall stations within the study area

QC	Station Number	RL (years)	QC	Station Number	RL (years)	QC	Station Number	RL (years)
C51A	0231161W	49	C51G	0201756W	28	C51K	0258458W	98
	0231247W	35		0201843W	29		0258467W	51
	0231279W	93		0230210W	34		0258474W	34
	0231395W	71	C51H	0229579W	40		0258581W	56
	0231761W	31		0229629W	47		0258624W	63
C51B	0231076W	44		0229654W	36		0258740W	51
	0231114W	35		0229737W	99		0258827W	44
	0231375W	47		0229862W	37		0258894W	99
	0231588W	37		0230011W	54		0259086W	25
	0231713W	56		0230027W	81		0259348W	71
	0232018W	86		0230048W	42	C51L	0257845W	85
C51C	0230764W	91		0230073W	76		0257878W	36
	0230816W	75		0230074W	24		0258182W	85
	0230774W	62		0230254W	39		0289796W	24
C51D	0231361W	64		0230349W	27	C51M	0256638W	78
	0231663W	26		0230466W	31		0257391W	66
	0231754W	55	C51J	0229344W	52	C52A	0232123W	88
	0232011W	41		0229555W	51		0232211W	88
	0261266W	36		0229556W	33		0232275W	94
	0261597W	53		0259390A	29		0232301W	38
	0261750W	55		0259743W	64		0232512W	35
C51E	0230542W	44		0260082W	33		0262353W	52
	0260660W	47		0260083W	33		0262479W	94
	0260715W	36	C51K	0228571W	68	C52B	0262129W	70
	0261146W	86		0228725W	52		0262314W	54
C51F	0229571W	50		0228783W	56		0262613A	76
	0229723W	47		0229124W	59		0262690W	47
	0230363W	41		0229215W	43		0262828W	30
	0260030W	80		0258079W	38	C52C	0262155W	32
	0230566W	39		0258157A	32		0262271W	28
	0230598W	30		0258164W	70		0262453W	24
	0230810W	93		0258213A	41		0262734W	43
C51G	0201361W	86		0258218W	50	C52D	0261789W	27
	0201370W	42		0258306W	68		0261890W	36
	0201373W	53		0258335W	50		0262247W	24
	0201482W	86		0258339W	51	C52E	0261722W	94
	0201492W	43		0258380W	63		0294052W	53
	0201637W	37		0258434W	61		0294233W	85

Table A.3: (continued)

QC	Station Number	RL (years)	QC	Station Number	RL (years)	QC	Station Number	RL (years)
C52E	0294417W	67	**C52K**	0259855W	43	**C52L**	0258812W	70
C52F	0261307A	25		0259881W	54		0259002W	45
	0261365W	73		0259887W	49		0259102W	31
	0261366W	39		0260004W	89		0259131W	30
	0261367W	46		0260126W	32		0259278W	67
	0261368W	88		0260163W	74		0290468W	33
	0261369W	47		0260314W	39		0290560W	82
	0261425W	46		0291075W	32		0290810W	64
	0261517W	37		0291077W	24		0290887W	45
	0261523W	94		0291148W	90	**Neighboring stations**	0229170W	46
	0261548W	26		0291174W	34		0232083W	58
	0261733W	70		0291178W	59		0232218W	36
C52G	0293204W	61		0291231W	42		0232522W	34
	0293403W	24		0291313W	44		0256453W	99
	0293514W	73		0291323W	29		0256631W	27
	0293568W	38		0291360W	44		0257655W	78
	0293597W	70		0291415W	46		0262694W	72
	0293622W	66		0291582W	39		0262886W	33
	0293700W	90		0291708W	47		0263057W	40
	0293792W	90		0291758W	33		0290032W	99
C52H	0261183W	95		0291899A	79		0290191W	50
	0261275W	60		0291899W	85		0290463W	49
	0292461W	90		0292051W	41		0290464A	75
	0292606W	40		0292089W	36		0291159W	34
	0292833W	47	**Neighbouring stations**	0200361W	33		0291245W	50
	0293007W	66		0200579W	28		0291392W	99
	0293106W	71		0200791W	44		0294500W	76
	0293339W	35		0200855W	24		0294651W	40
C52J	0292446W	35		0201020W	94		0325870W	42
	0261256W	36		0201376A	62		0326298W	43
	0260555W	50		0201376W	76		0327019W	29
	0260882W	56		0201701W	54		0327264W	80
	0261312W	63		0202366W	30		0327899W	53
	0260519W	66		0202575W	40			
	0260678W	88		0227811W	64			
C52K	0259578W	57		0228458W	75			
	0259609W	49		0228495W	92			
	0259727W	94		0228710W	39			

Table A.4: Thiessen weights at a QC level in tertiary drainage region C51

QC	Station	Thiessen weight	QC	Station	Thiessen weight	QC	Station	Thiessen weight
C51A	0231361W	0.0778	C51F	0230466W	0.0349	C51J	0259743W	0.1427
	0231247W	0.0548		0230254W	0.0580		0229737W	0.0172
	0231279W	0.0313		0230073W	0.0065		0229579W	0.1302
	0231395W	0.1148		0230011W	0.0547		0229555W	0.1760
	0231761W	0.1758		0260082W	0.1012		0229344W	0.1073
	0230816W	0.1498		0260030W	0.2311		0229215W	0.0444
	0231161W	0.0515		0229723W	0.1062		0260082W	0.1177
	0231076W	0.0824		0229571W	0.0621		0259887W	0.0186
	0230764W	0.0825		0230363W	0.3454		0259390A	0.1469
	0231375W	0.0977	C51G	0230349W	0.0405		0229723W	0.0742
	0231588W	0.0816		0201492W	0.0699		0229571W	0.0249
C51B	0202366W	0.1035		0200791W	0.0316	C51L	0258213A	0.0083
	0231076W	0.1085		0201020W	0.0019		0290191W	0.0241
	0232018W	0.0514		0200855W	0.0301		0289796W	0.3748
	0202575W	0.0613		0230210W	0.1151		0257391W	0.1304
	0230810W	0.0013		0201756W	0.0485		0258182W	0.1263
	0201843W	0.0162		0201701W	0.0146		0257878W	0.0932
	0232083W	0.0064		0201637W	0.0583		0257845W	0.1577
	0232011W	0.0054		0201482W	0.0660		0290468W	0.0583
	0231761W	0.0319		0201376W	0.0023		0290464A	0.0104
	0231713W	0.1360		0201376AW	0.0200		0290032W	0.0017
	0231588W	0.1390		0201373W	0.0234		0258218W	0.0149
	0231375W	0.1048		0201370W	0.1026	C51K	0258213A	0.0057
	0231161W	0.0161		0201361W	0.0756		0259743W	0.0265
	0231114W	0.1710		0230810W	0.0587		0258894W	0.0761
	0230774W	0.0274		0201843W	0.0481		0258474W	0.0358
	0230764W	0.0186	C51G	0231114W	0.0055		0257655W	0.0016
	0230816W	0.0010		0230774W	0.0334		0228495W	0.0044
C51C	0230542W	0.0832		0230598W	0.0575		0229170W	0.0006
	0230774W	0.0988		0230566W	0.0868		0229344W	0.0228
	0230764W	0.3195		0230048W	0.0026		0258380W	0.0474
	0230466W	0.1568		0230027W	0.0065		0258079W	0.0914
	0260660W	0.0542		0229862W	0.0004		0228571W	0.0443
	0231247W	0.0081	C51H	0229737W	0.0803		0228458W	0.0031
	0230816W	0.2795		0230349W	0.0675		0259348W	0.0986
C51D	0231754W	0.1406		0200791W	0.0276		0258827W	0.0442
	0261266W	0.0975		0200361W	0.0196		0258740W	0.0362
	0232011W	0.1267		0230210W	0.0202		0258624W	0.0374
	0231761W	0.0688		0230566W	0.0088		0229124W	0.1100
	0231395W	0.0221		0230466W	0.0658		0228783W	0.0373
	0231361W	0.1040		0230254W	0.0688		0228725W	0.0531
	0261750W	0.1262		0230074W	0.0282		0259609W	0.0013
	0261597W	0.2472		0230073W	0.0271		0259102W	0.0048
	0232218W	0.0457		0230048W	0.0709		0258581W	0.0410
	0232123W	0.0212		0230027W	0.1294		0258467W	0.0240
C51E	0230542W	0.2023		0230011W	0.0863		0258458W	0.0156
	0260715W	0.1873		0229862W	0.0608		0258434W	0.0145
	0260660W	0.1495		0229654W	0.1554		0258399W	0.0073
	0230363W	0.0096		0229579W	0.0543		0258339W	0.0213
	0261146AW	0.2797		0229555W	0.0138		0258306W	0.0078
	0231361W	0.0538		0229723W	0.0152		0258218W	0.0115
	0231247W	0.0474	C51M	0257878W	0.0536		0258164W	0.0463
	0230816W	0.0647		0257845W	0.0017		0258157A	0.0093
	0260555W	0.0056		0256453W	0.0001		0257878W	0.0186
				0256638W	0.2481			
				0257391W	0.5526			
				0257655W	0.1375			
				0258079W	0.0064			

Table A.5: Thiessen weights at a QC level in tertiary drainage region C52

QC	Station	Thiessen weight	QC	Station	Thiessen weight	QC	Station	Thiessen weight
C52A	0262690W	0.0099	C52F	0261789W	0.0320	C52J	0260519W	0.1919
	0231754W	0.0309		0261890W	0.0894		0261183W	0.0116
	0261890W	0.0983		0261597W	0.0467		0261266W	0.0215
	0232275W	0.0882		0293597A	0.0424		0260715W	0.0388
	0232218W	0.0164		0261733W	0.1312		0260678W	0.1016
	0232211W	0.1012		0261722W	0.0421		0261256W	0.1278
	0232123W	0.1411		0261548W	0.1130		0261146W	0.0261
	0262479W	0.0654		0261523W	0.2252		0261597W	0.0032
	0262353W	0.2056		0261517W	0.0935		0261523W	0.0329
	0232512W	0.1230		0261426W	0.0568		0261312W	0.0683
	0232301W	0.0836		0261368W	0.0186		0260882W	0.1553
	0262314W	0.0365		0261367W	0.0589		0292446W	0.1035
C52B	0262690W	0.1477		0261366W	0.0098		0292089W	0.0073
	0262828W	0.1274		0261365W	0.0095		0260555W	0.0911
	0262479W	0.0603		0261307A	0.0241		0260314W	0.0191
	0262353W	0.0773		0261275W	0.0068	C52K	0260519W	0.0023
	0232512W	0.0046	C52G	0327899W	0.0065		0325870W	0.0365
	0262734W	0.0376		0293792W	0.1359		0292461W	0.0606
	0262613A	0.1985		0293106W	0.0953		0291582W	0.0873
	0262314W	0.2507		0261183W	0.0012		0291075W	0.0261
	0262247W	0.0047		0293597A	0.0140		0259727W	0.0323
	0262129W	0.0911		0293568W	0.0657		0230542W	0.0004
C52C	0262694W	0.0648		0293204W	0.1784		0259743W	0.0086
	0294417W	0.0481		0261426W	0.0136		0260082W	0.0128
	0262734W	0.0665		0261365W	0.0255		0259887W	0.0361
	0262613A	0.0883		0261275W	0.0074		0292446W	0.0390
	0262453W	0.3312		0293514W	0.0570		0260555W	0.0049
	0262314W	0.0072		0293339W	0.0581		0260314W	0.0462
	0262271W	0.2751		0294052W	0.0487		0260163W	0.0363
	0262129W	0.1189		0293700W	0.1281		0291899W	0.0395
C52D	0261789W	0.2265		0293622W	0.1646		0291899A	0.0303
	0261890W	0.3669	C52H	0327264W	0.0563		0260126W	0.0449
	0262314W	0.00002		0293106W	0.0807		0260004W	0.0188
	0262129W	0.2311		0292461W	0.1138		0259881W	0.0279
	0261733W	0.1512		0261183W	0.1116		0259609W	0.0339
	0261722W	0.0243		0293204W	0.0081		0259578W	0.0290
C52E	0293792W	0.0089		0261312W	0.0023		0291360W	0.0468
	0261789W	0.0209		0261275W	0.0279		0291178W	0.0104
	0294417W	0.1247		0260882W	0.0157		0291174W	0.0049
	0293597A	0.0625		0292446W	0.0719		0291148W	0.0023
	0262271W	0.0547		0292606W	0.0643		0291323W	0.0459
	0261722W	0.2139		0293514W	0.0554		0291245W	0.0048
	0294233W	0.2104		0293339W	0.0999		0291231W	0.0292
	0294052W	0.2642		0293007W	0.1522		0291159W	0.0217
	0293622W	0.0071		0292833W	0.1398		0292051W	0.1019
	0294500W	0.0328	C52L	0291178W	0.0170		0291708W	0.0736
C52L	0258213A	0.0072		0291174W	0.0510		0291392W	0.0047
	0291075W	0.0523		0291148W	0.0157	C52L	0258399W	0.0406
	0259002W	0.0523		0290810W	0.0865		0258182W	0.0269
	0259348W	0.0476		0259278W	0.0998		0290887W	0.0474
	0258827W	0.0106		0259102W	0.0900		0290560W	0.1422
	0259609W	0.0289		0258812W	0.0717		0290464AW	0.0145
	0259578W	0.0179		0258581W	0.0077		0290463W	0.0007
	0291360W	0.0191		0258458W	0.0431		0291231W	0.0095

Table A.6: Geographically-centred sample ARF values at a QC level in the C51 tertiary drainage region

	RI (years)	Storm duration (hours)					
		1	**8**	**16**	**24**	**72**	**168**
C51A	**2**	0.643	0.643	0.643	0.643	0.815	0.866
	5	0.724	0.724	0.724	0.724	0.851	0.892
	10	0.768	0.768	0.768	0.768	0.870	0.906
	20	0.805	0.805	0.805	0.805	0.886	0.919
	50	0.848	0.848	0.848	0.848	0.903	0.934
	100	0.877	0.877	0.877	0.877	0.915	0.944
	200	0.904	0.904	0.904	0.904	0.925	0.954
C51B	**2**	0.660	0.660	0.660	0.660	0.827	0.863
	5	0.713	0.713	0.713	0.713	0.853	0.888
	10	0.746	0.746	0.746	0.746	0.863	0.901
	20	0.777	0.777	0.777	0.777	0.871	0.912
	50	0.816	0.816	0.816	0.816	0.878	0.924
	100	0.846	0.846	0.846	0.846	0.882	0.932
	200	0.875	0.875	0.875	0.875	0.884	0.940
C51C	**2**	0.685	0.685	0.685	0.685	0.841	0.878
	5	0.730	0.730	0.730	0.730	0.873	0.910
	10	0.755	0.755	0.755	0.755	0.892	0.932
	20	0.775	0.775	0.775	0.775	0.909	0.955
	50	0.800	0.800	0.800	0.800	0.929	0.984
	100	0.816	0.816	0.816	0.816	0.944	1.006
	200	0.832	0.832	0.832	0.832	0.958	1.029
C51D	**2**	0.664	0.664	0.664	0.664	0.837	0.885
	5	0.731	0.731	0.731	0.731	0.881	0.919
	10	0.784	0.784	0.784	0.784	0.909	0.943
	20	0.843	0.843	0.843	0.843	0.934	0.965
	50	0.930	0.930	0.930	0.930	0.967	0.995
	100	1.003	1.003	1.003	1.003	0.991	1.018
	200	1.085	1.085	1.085	1.085	1.015	1.042
C51E	**2**	0.692	0.692	0.692	0.692	0.851	0.882
	5	0.735	0.735	0.735	0.735	0.902	0.923
	10	0.752	0.752	0.752	0.752	0.926	0.944
	20	0.763	0.763	0.763	0.763	0.945	0.960
	50	0.772	0.772	0.772	0.772	0.964	0.978
	100	0.776	0.776	0.776	0.776	0.976	0.990
	200	0.778	0.778	0.778	0.778	0.987	1.001
C51F	**2**	0.683	0.683	0.683	0.683	0.811	0.864
	5	0.737	0.737	0.737	0.737	0.846	0.899
	10	0.770	0.770	0.770	0.770	0.868	0.917
	20	0.801	0.801	0.801	0.801	0.888	0.932
	50	0.840	0.840	0.840	0.840	0.914	0.949
	100	0.870	0.870	0.870	0.870	0.933	0.961
	200	0.900	0.900	0.900	0.900	0.953	0.972

Table A.6: (continued)

	RI (years)	Storm duration (hours)					
		1	**8**	**16**	**24**	**72**	**168**
C51G	**2**	0.647	0.647	0.647	0.647	0.794	0.846
	5	0.671	0.671	0.671	0.671	0.837	0.872
	10	0.678	0.678	0.678	0.678	0.862	0.890
	20	0.680	0.680	0.680	0.680	0.883	0.908
	50	0.679	0.679	0.679	0.679	0.909	0.931
	100	0.676	0.676	0.676	0.676	0.928	0.950
	200	0.673	0.673	0.673	0.673	0.946	0.968
C51H	**2**	0.645	0.645	0.645	0.645	0.823	0.872
	5	0.692	0.692	0.692	0.692	0.865	0.903
	10	0.713	0.713	0.713	0.713	0.881	0.914
	20	0.730	0.730	0.730	0.730	0.892	0.920
	50	0.748	0.748	0.748	0.748	0.901	0.924
	100	0.759	0.759	0.759	0.759	0.906	0.925
	200	0.769	0.769	0.769	0.769	0.909	0.924
C51J	**2**	0.611	0.611	0.611	0.611	0.816	0.875
	5	0.677	0.677	0.677	0.677	0.845	0.911
	10	0.716	0.716	0.716	0.716	0.853	0.930
	20	0.750	0.750	0.750	0.750	0.856	0.945
	50	0.791	0.791	0.791	0.791	0.857	0.963
	100	0.821	0.821	0.821	0.821	0.855	0.976
	200	0.849	0.849	0.849	0.849	0.852	0.988
C51K	**2**	0.610	0.610	0.610	0.610	0.786	0.833
	5	0.685	0.685	0.685	0.685	0.839	0.873
	10	0.734	0.734	0.734	0.734	0.867	0.899
	20	0.782	0.782	0.782	0.782	0.891	0.924
	50	0.847	0.847	0.847	0.847	0.917	0.958
	100	0.898	0.898	0.898	0.898	0.935	0.984
	200	0.952	0.952	0.952	0.952	0.950	1.011
C51L	**2**	0.693	0.693	0.693	0.693	0.847	0.886
	5	0.739	0.739	0.739	0.739	0.884	0.918
	10	0.775	0.775	0.775	0.775	0.903	0.941
	20	0.813	0.813	0.813	0.813	0.919	0.964
	50	0.868	0.868	0.868	0.868	0.937	0.995
	100	0.913	0.913	0.913	0.913	0.949	1.019
	200	0.961	0.961	0.961	0.961	0.960	1.045
C51M	**2**	0.729	0.729	0.729	0.729	0.847	0.875
	5	0.785	0.785	0.785	0.785	0.874	0.906
	10	0.826	0.826	0.826	0.826	0.892	0.926
	20	0.866	0.866	0.866	0.866	0.909	0.945
	50	0.921	0.921	0.921	0.921	0.931	0.970
	100	0.965	0.965	0.965	0.965	0.949	0.989
	200	1.010	1.010	1.010	1.010	0.966	1.008

Table A.7: Geographically-centred sample ARF values at a QC level in the C52 tertiary drainage region

	RI (years)	Storm duration (hours)					
		1	8	16	24	72	168
C52A	2	0.729	0.729	0.729	0.729	0.869	0.911
	5	0.780	0.780	0.780	0.780	0.901	0.937
	10	0.812	0.812	0.812	0.812	0.922	0.949
	20	0.842	0.842	0.842	0.842	0.942	0.958
	50	0.881	0.881	0.881	0.881	0.968	0.966
	100	0.911	0.911	0.911	0.911	0.988	0.971
	200	0.942	0.942	0.942	0.942	1.008	0.975
C52B	2	0.722	0.722	0.722	0.722	0.857	0.889
	5	0.771	0.771	0.771	0.771	0.899	0.909
	10	0.806	0.806	0.806	0.806	0.927	0.925
	20	0.842	0.842	0.842	0.842	0.954	0.941
	50	0.889	0.889	0.889	0.889	0.989	0.964
	100	0.927	0.927	0.927	0.927	1.015	0.981
	200	0.965	0.965	0.965	0.965	1.041	1.000
C52C	2	0.729	0.729	0.729	0.729	0.853	0.901
	5	0.764	0.764	0.764	0.764	0.883	0.924
	10	0.775	0.775	0.775	0.775	0.900	0.937
	20	0.781	0.781	0.781	0.781	0.915	0.948
	50	0.783	0.783	0.783	0.783	0.934	0.962
	100	0.783	0.783	0.783	0.783	0.948	0.971
	200	0.780	0.780	0.780	0.780	0.961	0.980
C52D	2	0.785	0.785	0.785	0.785	0.887	0.927
	5	0.795	0.795	0.795	0.795	0.921	0.951
	10	0.799	0.799	0.799	0.799	0.944	0.966
	20	0.801	0.801	0.801	0.801	0.966	0.980
	50	0.802	0.802	0.802	0.802	0.995	0.998
	100	0.802	0.802	0.802	0.802	1.018	1.012
	200	0.801	0.801	0.801	0.801	1.041	1.025
C52E	2	0.741	0.741	0.741	0.741	0.829	0.880
	5	0.784	0.784	0.784	0.784	0.878	0.910
	10	0.805	0.805	0.805	0.805	0.906	0.928
	20	0.822	0.822	0.822	0.822	0.930	0.947
	50	0.841	0.841	0.841	0.841	0.960	0.970
	100	0.853	0.853	0.853	0.853	0.981	0.988
	200	0.864	0.864	0.864	0.864	1.001	1.006
C52F	2	0.741	0.741	0.741	0.741	0.867	0.903
	5	0.791	0.791	0.791	0.791	0.897	0.927
	10	0.816	0.816	0.816	0.816	0.912	0.944
	20	0.836	0.836	0.836	0.836	0.923	0.962
	50	0.857	0.857	0.857	0.857	0.935	0.985
	100	0.871	0.871	0.871	0.871	0.942	1.004
	200	0.882	0.882	0.882	0.882	0.948	1.023

Table A.7: (continued)

	RI (years)	Storm duration (hours)					
		1	8	16	24	72	168
C52G	**2**	0.683	0.683	0.683	0.683	0.833	0.873
	5	0.717	0.717	0.717	0.717	0.877	0.909
	10	0.734	0.734	0.734	0.734	0.900	0.930
	20	0.748	0.748	0.748	0.748	0.920	0.949
	50	0.764	0.764	0.764	0.764	0.943	0.973
	100	0.774	0.774	0.774	0.774	0.958	0.990
	200	0.783	0.783	0.783	0.783	0.973	1.007
C52H	**2**	0.636	0.636	0.636	0.636	0.795	0.839
	5	0.663	0.663	0.663	0.663	0.839	0.884
	10	0.673	0.673	0.673	0.673	0.857	0.911
	20	0.679	0.679	0.679	0.679	0.868	0.934
	50	0.685	0.685	0.685	0.685	0.878	0.962
	100	0.687	0.687	0.687	0.687	0.884	0.983
	200	0.689	0.689	0.689	0.689	0.887	1.003
C52J	**2**	0.686	0.686	0.686	0.686	0.821	0.869
	5	0.729	0.729	0.729	0.729	0.866	0.902
	10	0.749	0.749	0.749	0.749	0.895	0.923
	20	0.766	0.766	0.766	0.766	0.923	0.941
	50	0.784	0.784	0.784	0.784	0.960	0.964
	100	0.795	0.795	0.795	0.795	0.987	0.980
	200	0.805	0.805	0.805	0.805	1.013	0.996
C52K	**2**	0.615	0.615	0.615	0.615	0.808	0.843
	5	0.658	0.658	0.658	0.658	0.846	0.882
	10	0.699	0.699	0.699	0.699	0.864	0.905
	20	0.747	0.747	0.747	0.747	0.880	0.926
	50	0.820	0.820	0.820	0.820	0.898	0.953
	100	0.884	0.884	0.884	0.884	0.910	0.974
	200	0.955	0.955	0.955	0.955	0.921	0.994
C52L	**2**	0.581	0.581	0.581	0.581	0.784	0.851
	5	0.616	0.616	0.616	0.616	0.830	0.894
	10	0.645	0.645	0.645	0.645	0.860	0.913
	20	0.678	0.678	0.678	0.678	0.888	0.927
	50	0.725	0.725	0.725	0.725	0.926	0.941
	100	0.764	0.764	0.764	0.764	0.955	0.949
	200	0.807	0.807	0.807	0.807	0.985	0.956

Table A.8: ARF estimation results at a catchment level

Catchment (A, km²)	Tc (hours)	RI (years)	Design rainfall depth (mm)	Rainfall intensity (mm.h^{-1})	Avg. MAP (mm)	ARF (%)										
						Fig. (2.4)	Eq. (2.5)	Fig. (2.5)	Eq. (2.7)	Fig. (2.6)	Fig. (2.7)	Eq. (2.8)	Fig. (2.8)	Eq. (2.8a)	Eq. (2.10)	Eq. (5.1)
C5H022 (38)	1.6	10	50	31.2	563	95.5	92.3		76.9	87.5	96.0	98.2	95.0	95.1	96.1	78.5
		20	58	36.1		95.0	91.1									79.1
		50	69	43.0		94.0	89.5									80.8
		100	78	48.5		93.0	88.2									83.7
		200	87	54.3		92.0	86.9									89.5
C5R005 (116)	3.5	10	72	20.6	563	94.0	88.2		72.5	87.0	92.5	91.0	92.0	92.2	92.7	79.0
		20	84	23.9		92.0	86.4									79.6
		50	100	28.7		91.0	84.0									81.3
		100	114	32.4		90.0	82.0									84.2
		200	128	36.4		89.0	80.1									90.0
C5H054 (688)	1.9	10	78	4.6	502	96.0	89.6	74.0	73.6	88.8	89.0	87.3	88.5	88.8	86.9	74.1
		20	89	5.3		96.0	88.1									74.6
		50	105	6.2		95.5	86.1									76.4
		100	118	7.0		95.5	84.6									79.3
		200	131	7.7		95.0	83.1									85.1
C5R001 (922)	2.3	10	85	4.0	473		88.8	74.5	74.7	89.0	89.0	87.4	87.5	88.1	85.9	82.3
		20	98	4.6			87.1									83.2
		50	117	5.5			84.8									85.6
		100	132	6.2			83.1									89.6
		200	147	6.9			81.2									97.8
C5R003 (937)	13.9	10	81	5.8	521		83.8	70.5	70.4	87.4	86.5	84.8	86.0	86.0	85.8	77.2
		20	94	6.7			81.5									77.7
		50	111	8.0			78.4									79.5
		100	126	9.0			76.0									82.4
		200	138	10.0			73.9									88.1
C5H003 (1 650)	18.3	10	80	4.4	543		81.4	69.5	71.1	87.0	88.5	84.0	83.5	83.6	83.8	83.1
		20	93	5.1			78.8									83.7
		50	109	6.0			75.6									85.4
		100	122	6.7			73.1									88.3
		200	135	7.4			70.7									94.1

Table A.8: (continued)

Catchment area (km^2)	*Tc* (hours)	RI (years)	Design rainfall depth (mm)	Rainfall intensity (mm.h^{-1})	Avg. MAP (mm)	ARF (%)										
						Fig. (2.4)	Eq. (2.5)	Fig. (2.5)	Eq. (2.7)	Fig. (2.6)	Fig. (2.7)	Eq. (2.8)	Fig. (2.8)	Eq. (2.8a)	Eq. (2.10)	Eq. (5.1)
C5H012 (2 366)	20.2	10	77	3.8	434		79.0	67.0	71.0	86.5	84.5	83.2	81.5	81.7	82.5	84.2
		20	89	4.4			76.1									85.0
		50	106	5.3			72.2									87.4
		100	120	5.9			69.3									91.5
		200	134	6.6			66.3									98.5
C5H015 (6 009)	43	10	93	2.2	505		75.9	64.5	81.3	87.0	85.0	86.1	79.0	79.2	78.9	95.3
		20	108	2.5			72.8									95.9
		50	127	3.0			68.7									97.6
		100	142	3.3			65.8									100.5
		200	157	3.7			62.9									106.3
C5R004 (6 331)	47.9	10	90	1.9	505		77.9	66.0	83.4	87.0	85.0	86.9	79.5	79.4	78.7	97.0
		20	104	2.2			75.0									97.5
		50	122	2.6			71.3									99.3
		100	137	2.9			68.5									102.2
		200	152	3.2			65.8									107.9
C5R002 (10 260)	50.5	10	85	1.7	406		72.5	60.5	85.3			86.1		76.1	76.7	
		20	98	1.9			68.9									
		50	117	2.3			64.2									
		100	132	2.6			60.6									
		200	147	2.9			57.1									
C5H018 (17 360)	99.6	10	115	1.2	448		71.7		106.7			91.9		76.1	74.5	
		20	133	1.3			68.1									
		50	156	1.6			63.7									
		100	174	1.8			60.4									
		200	193	1.9			57.3									
C5H016 (33 277)	111.1	10	107	1.0	417		63.3		117.8			92.2		71.5	71.6	
		20	123	1.1			59.0									
		50	145	1.3			53.7									
		100	162	1.5			49.9									
		200	180	1.6			46.3									

APPENDIX B: GRAPHICAL INFORMATION AND RESULTS

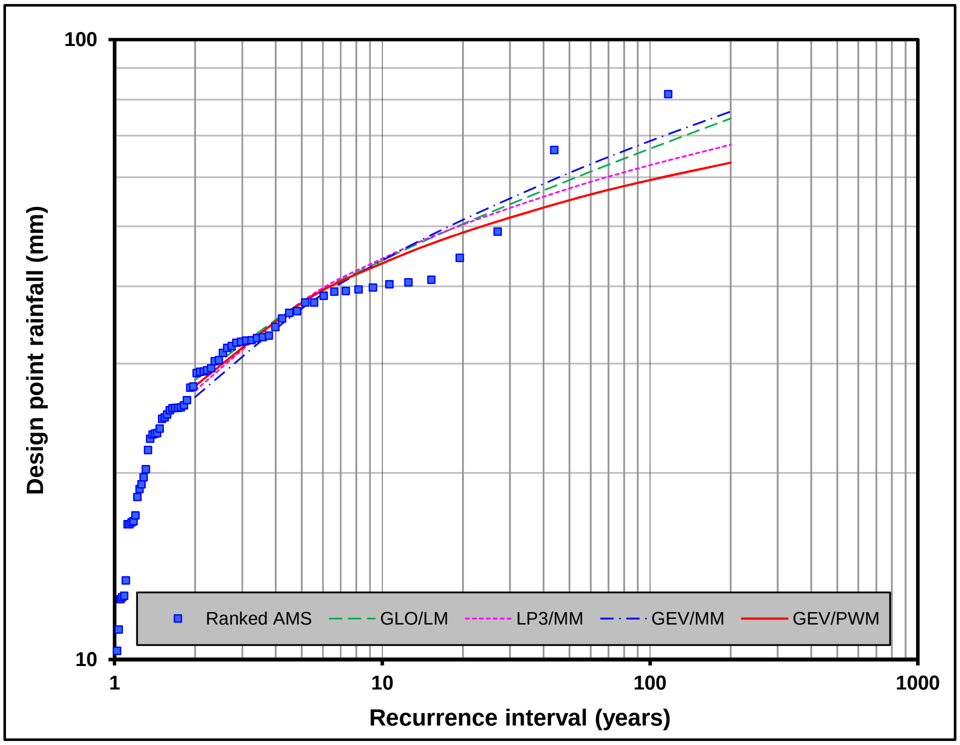

Figure B.1: 1-hour Probability distribution for design point rainfall in QC C51M

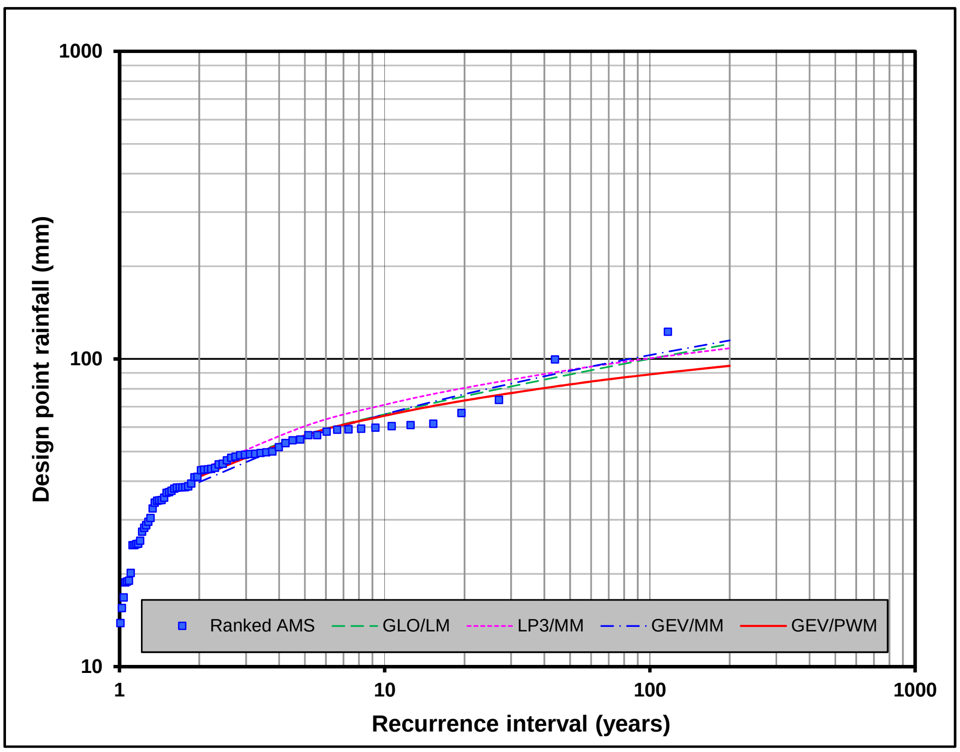

Figure B.2: 8-hour Probability distribution for design point rainfall in QC C51M

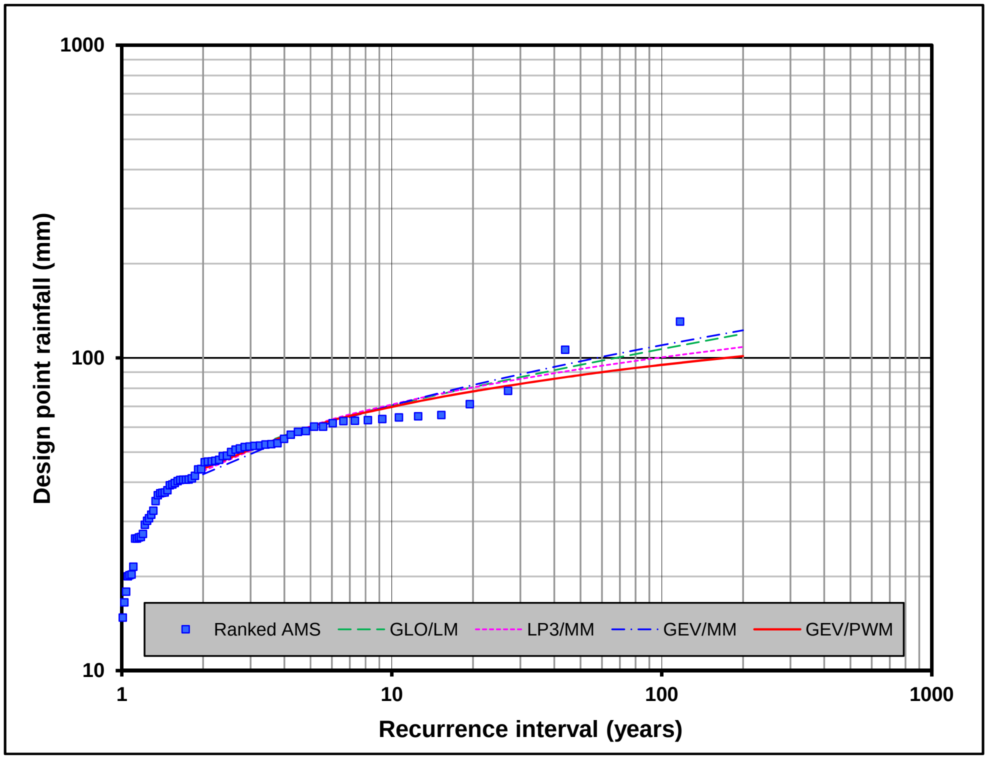

Figure B.3: 16-hour Probability distribution for design point rainfall in QC C51M

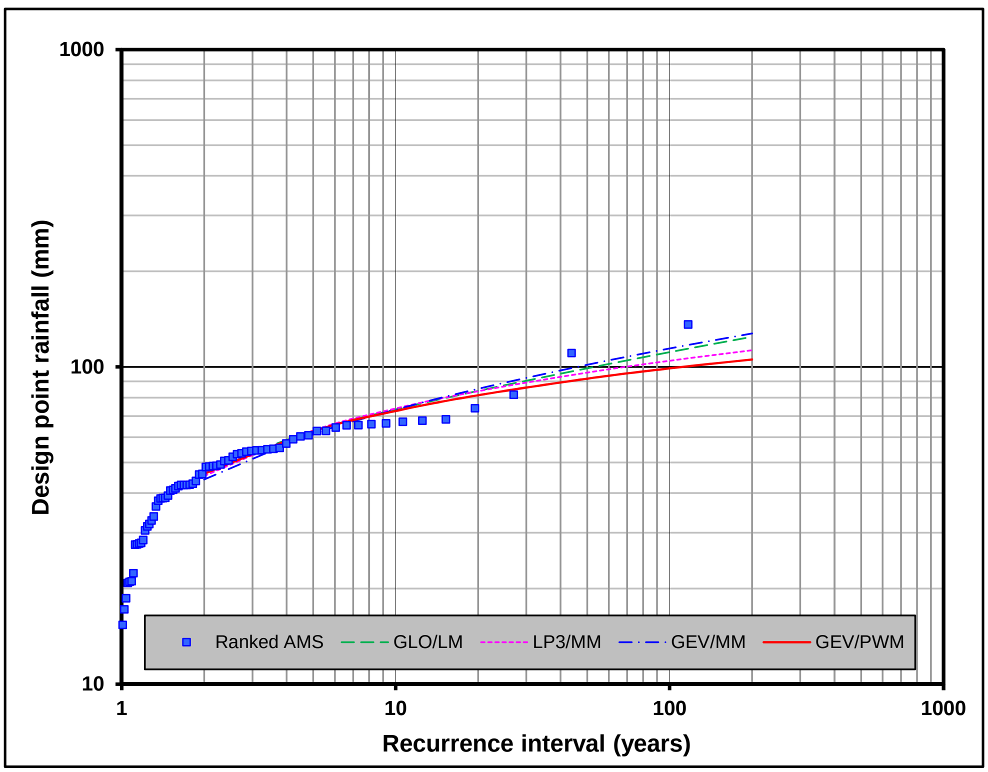

Figure B.4: 24-hour Probability distribution for design point rainfall in QC C51M

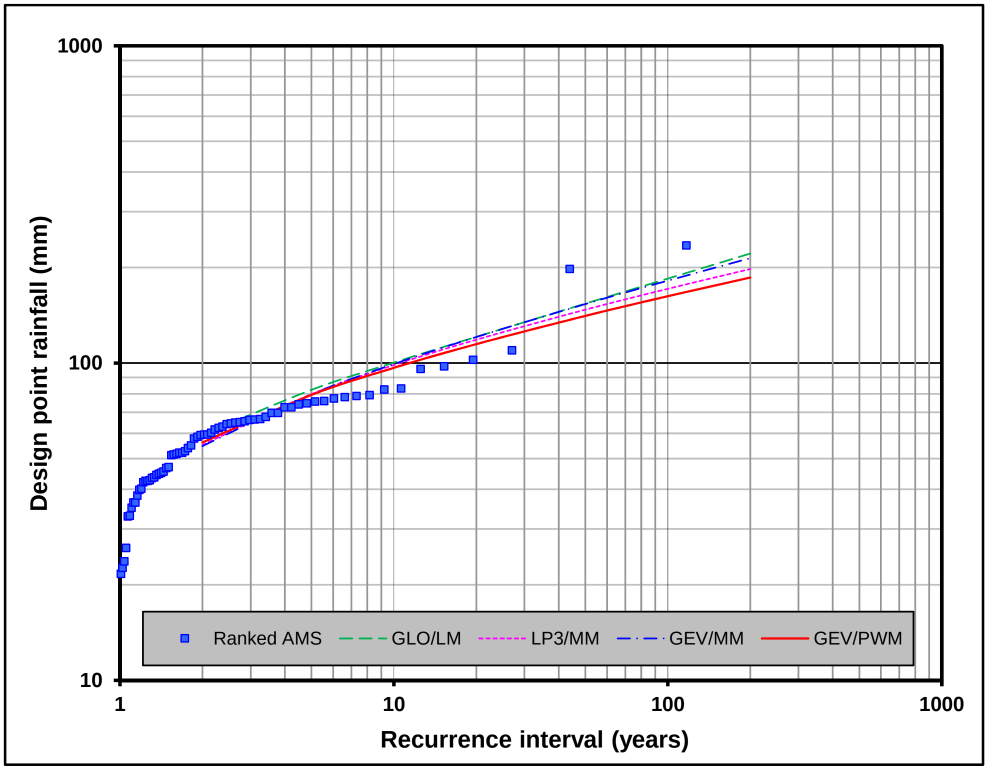

Figure B.5: 72-hour Probability distribution for design point rainfall in QC C51M

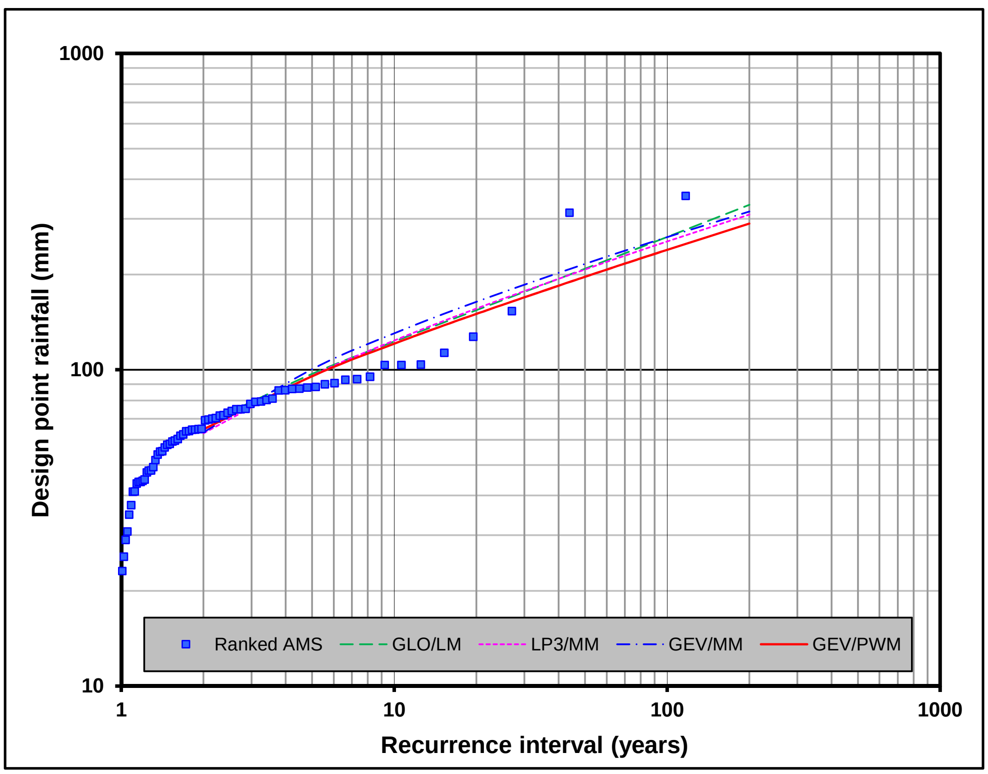

Figure B.6: 168-hour Probability distribution for design point rainfall in QC C51M

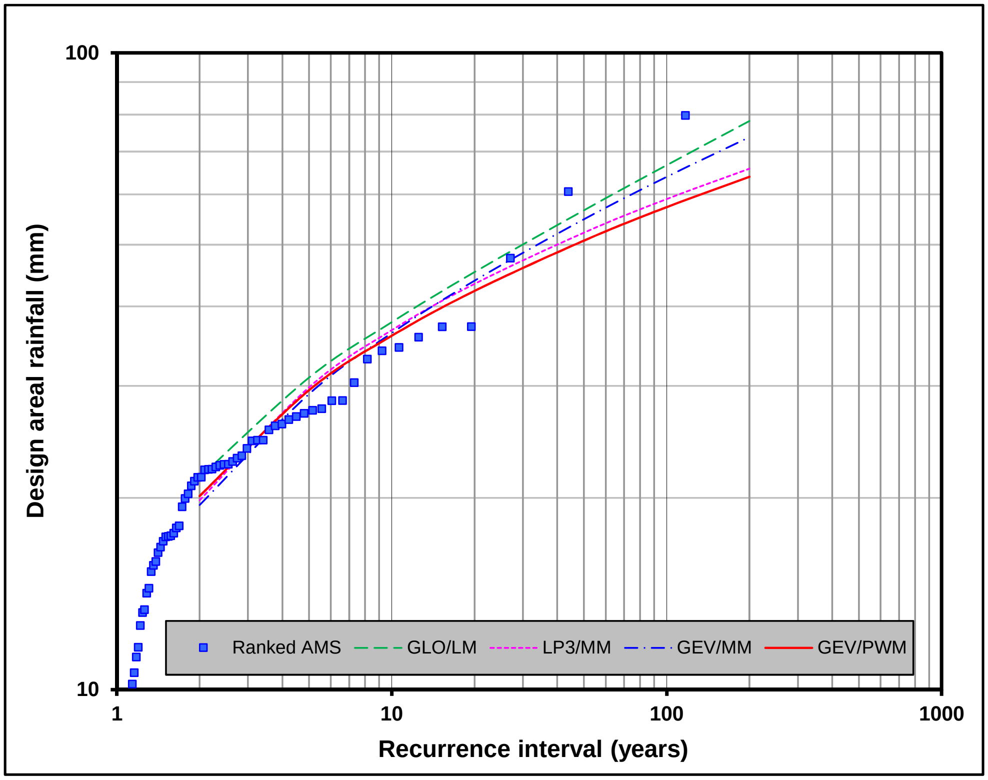

Figure B.7: 1-hour Probability distribution for areal design rainfall in QC C51M

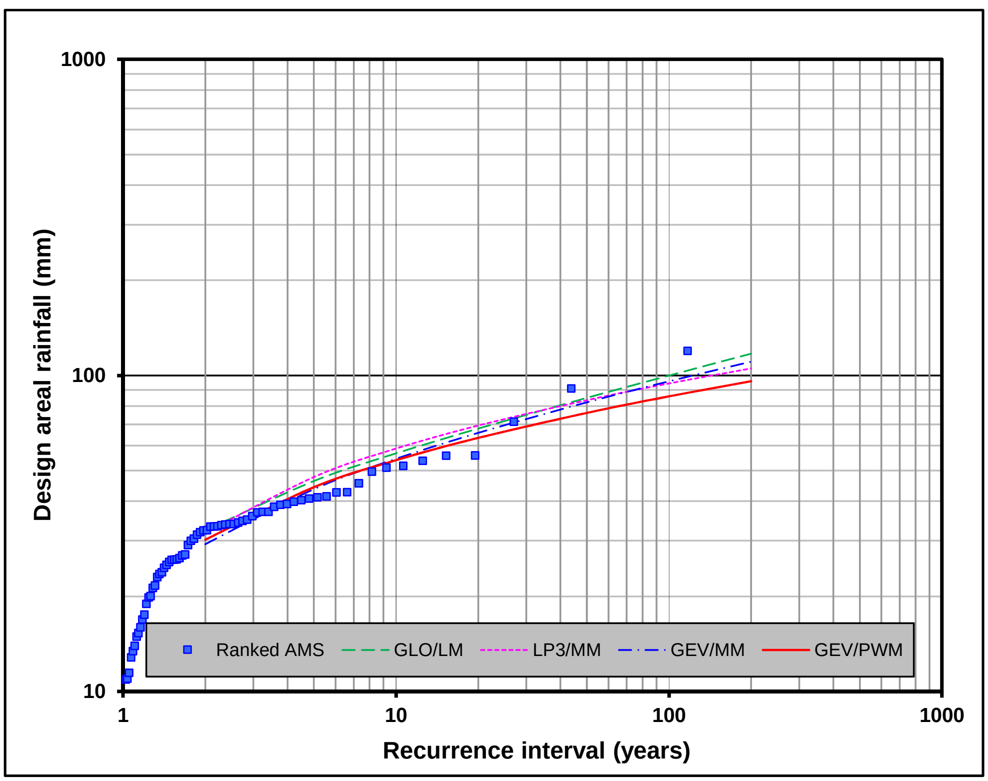

Figure B.8: 8-hour Probability distribution for areal design rainfall in QC C51M

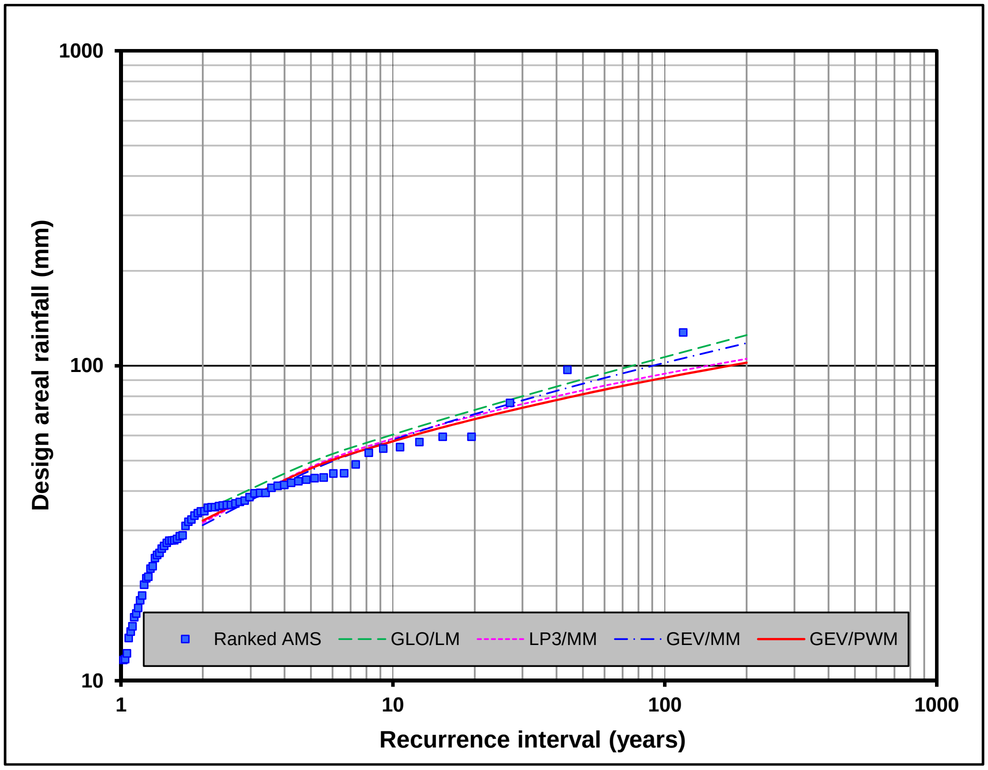

Figure B.9: 16-hour Probability distribution for areal design rainfall in QC C51M

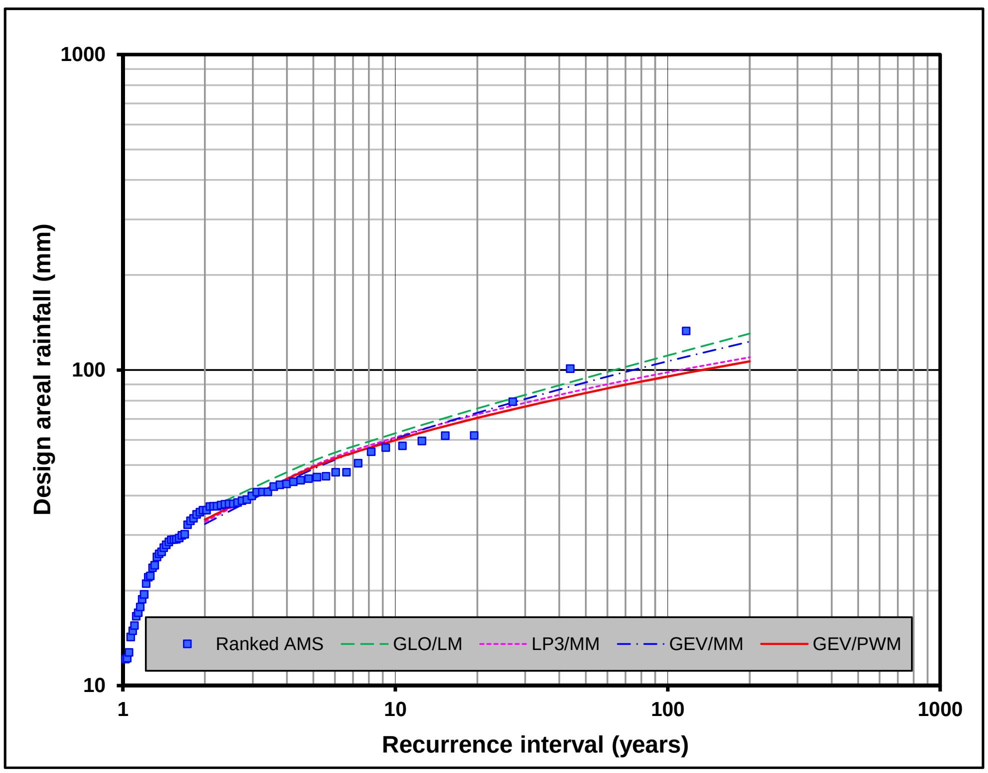

Figure B.10: 24-hour Probability distribution for areal design rainfall in QC C51M

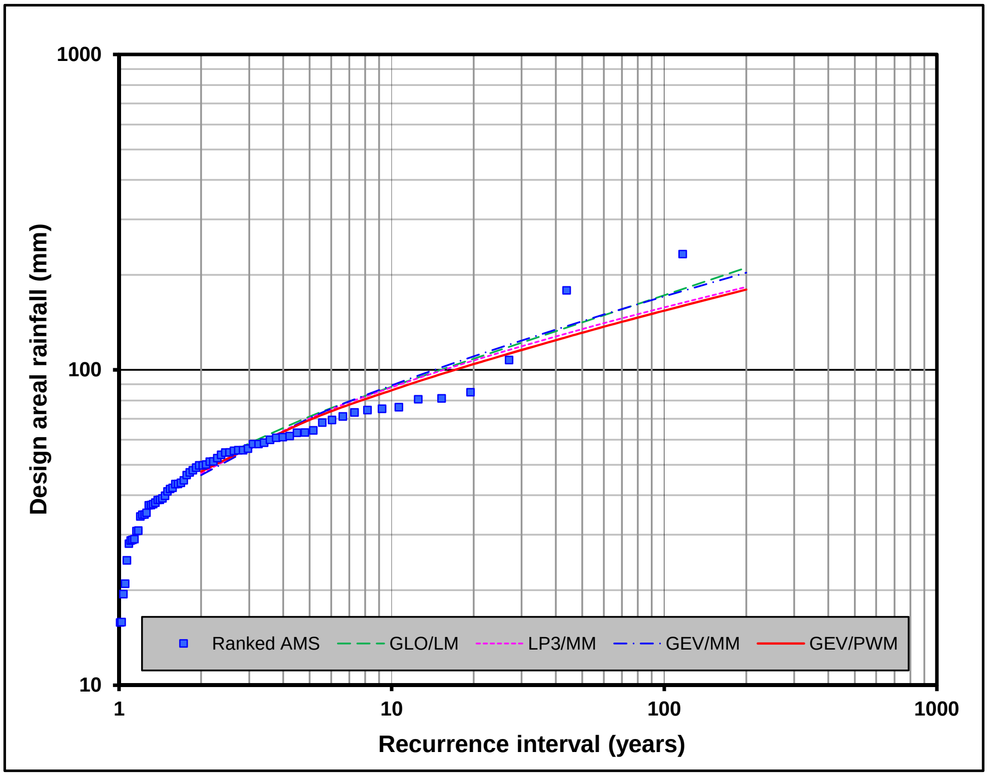

Figure B.11: 72-hour Probability distribution for areal design rainfall in QC C51M

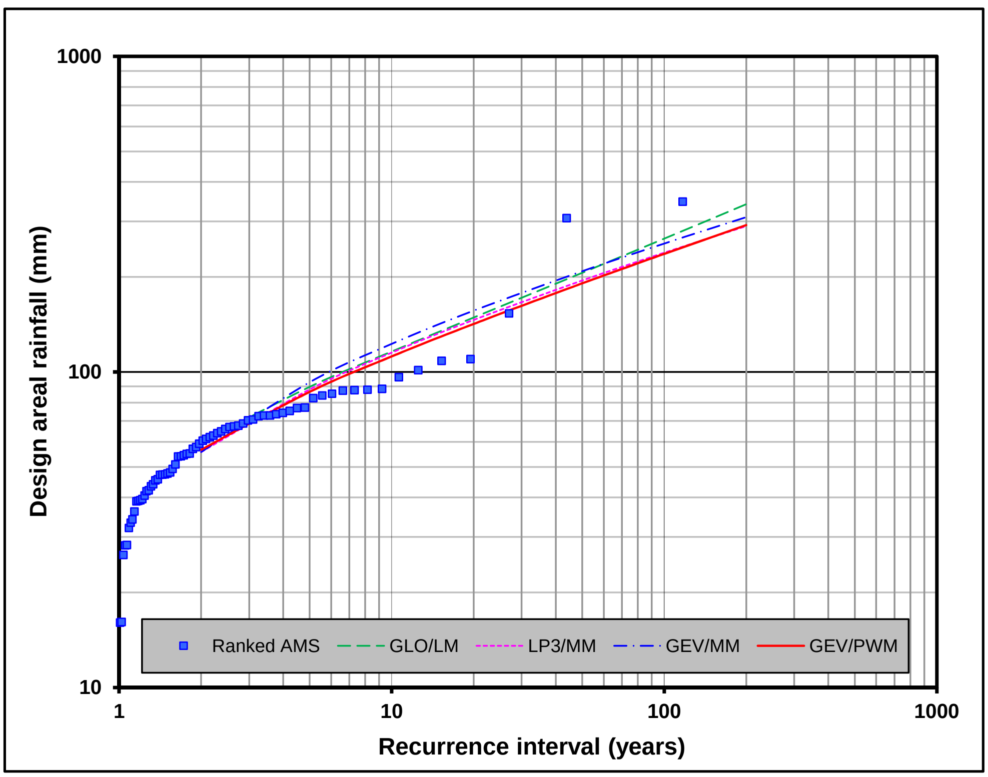

Figure B.12: 168-hour Probability distribution for areal design rainfall in QC C51M

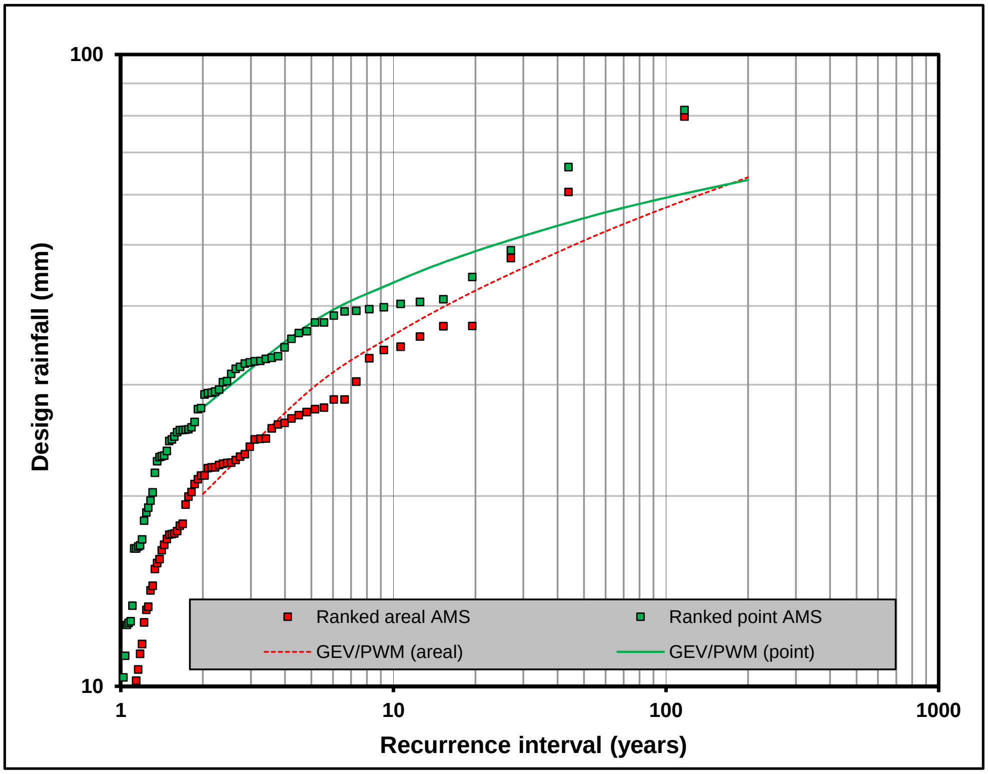

Figure B.13: 1-hour Point and areal design rainfall in QC C51M

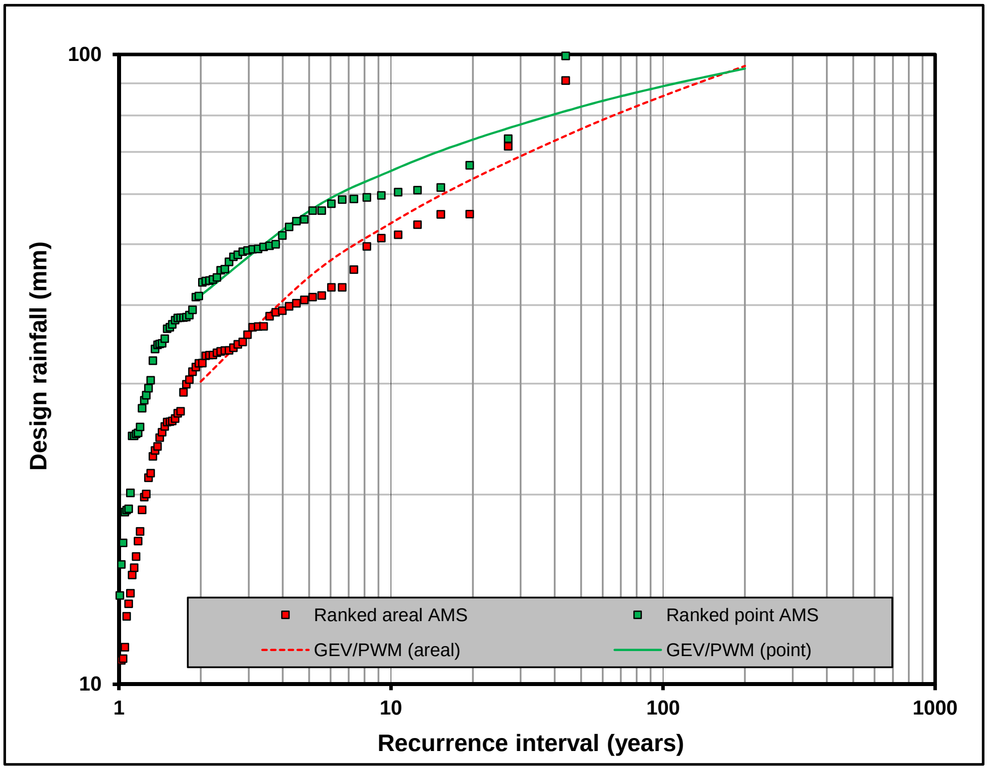

Figure B.14: 8-hour Point and areal design rainfall in QC C51M

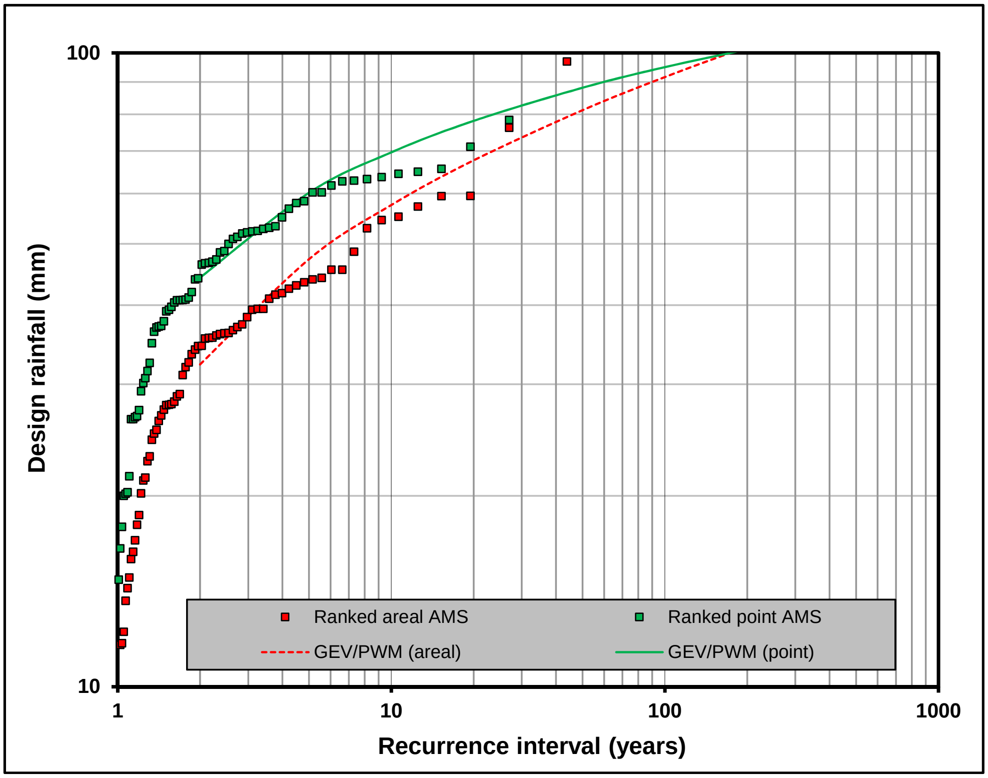

Figure B.15: 16-hour Point and areal design rainfall in QC C51M

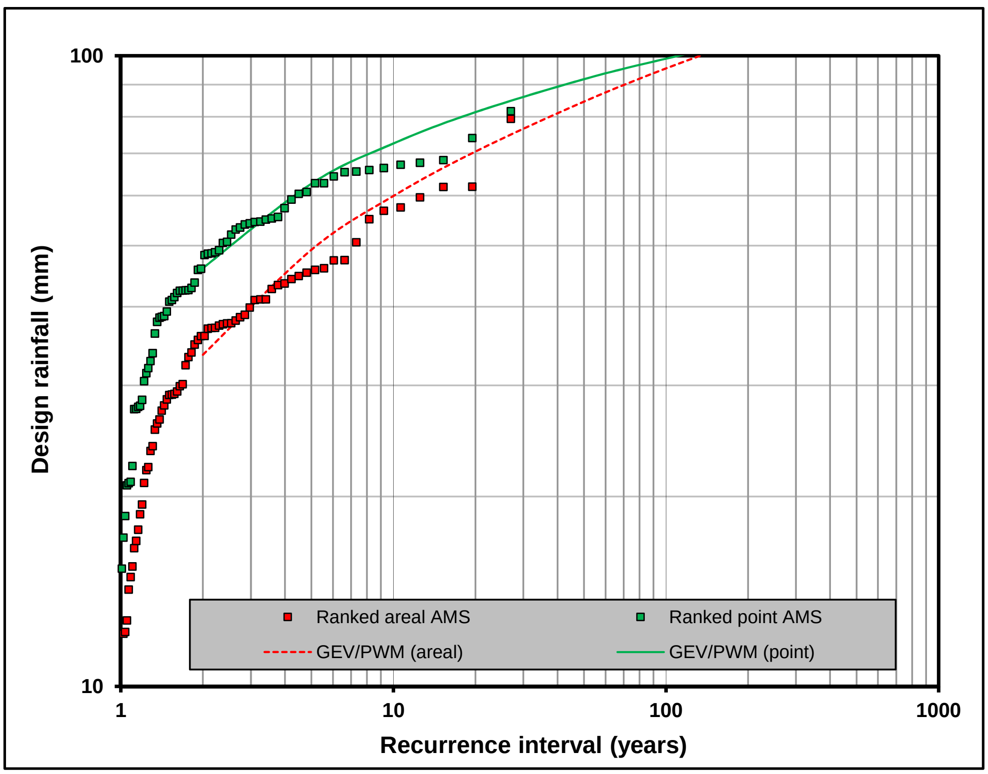

Figure B.16: 24-hour Point and areal design rainfall in QC C51M

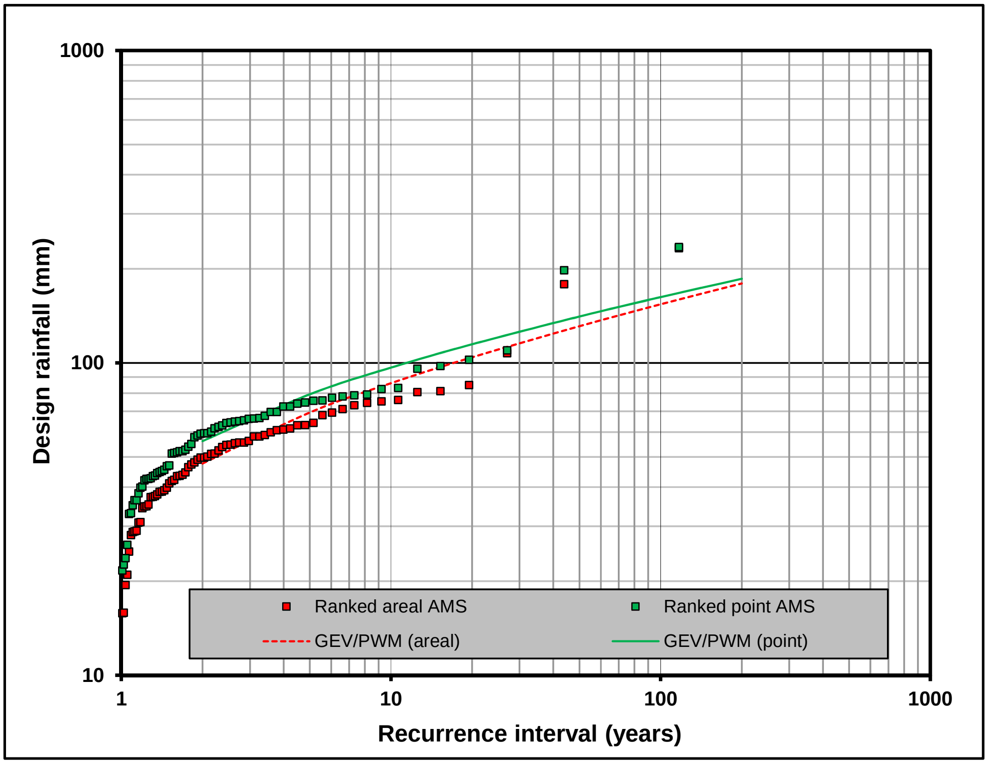

Figure B.17: 72-hour Point and areal design rainfall in QC C51M

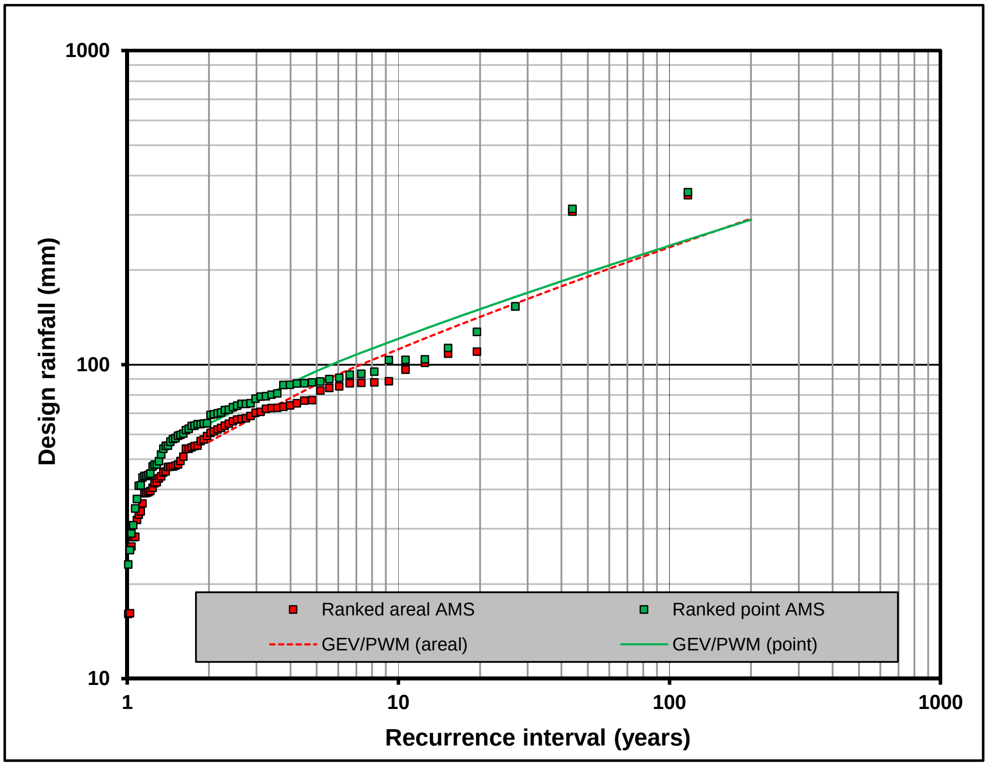

Figure B.18: 168-hour Point and areal design rainfall in QC C51M

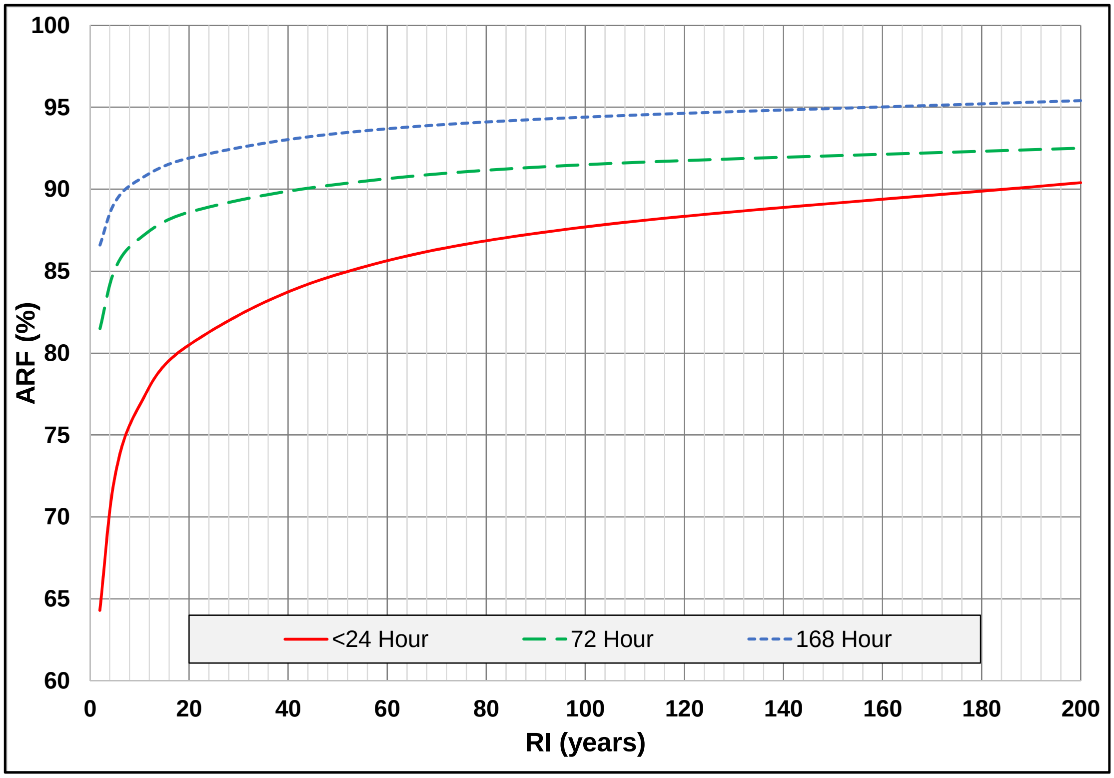

Figure B.19: Geographically-centred ARFs with corresponding RIs in QC C51A

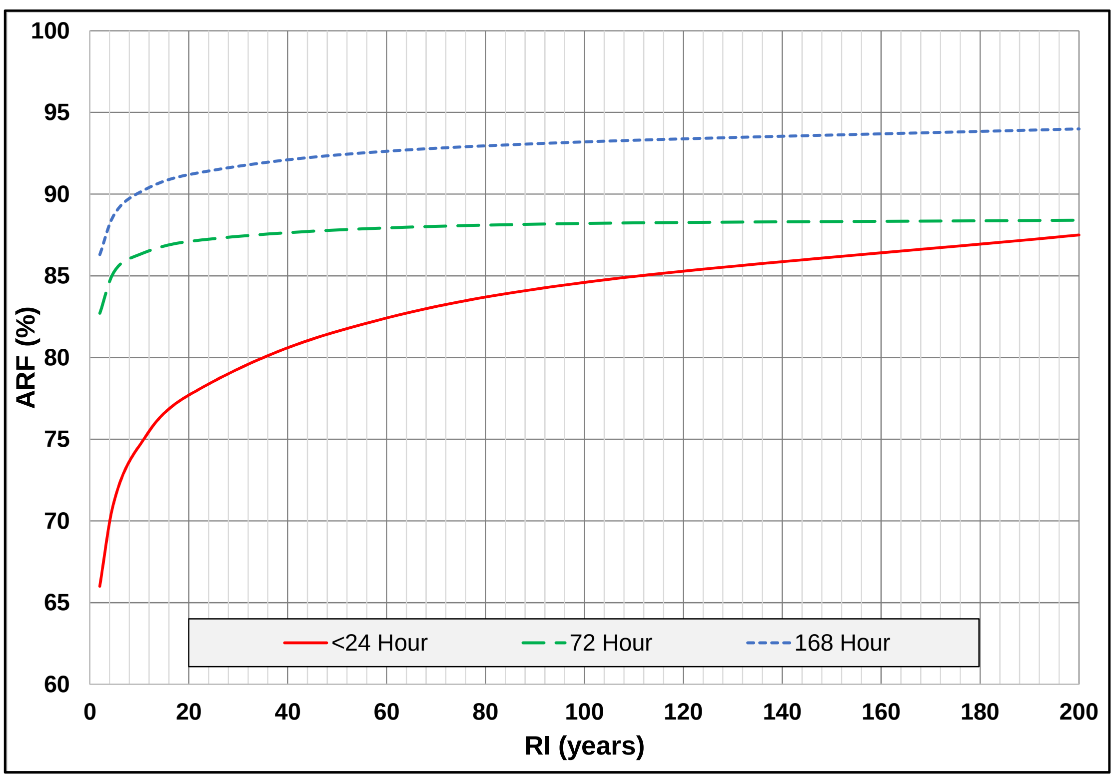

Figure B.20: Geographically-centred ARFs with corresponding RIs in QC C51B

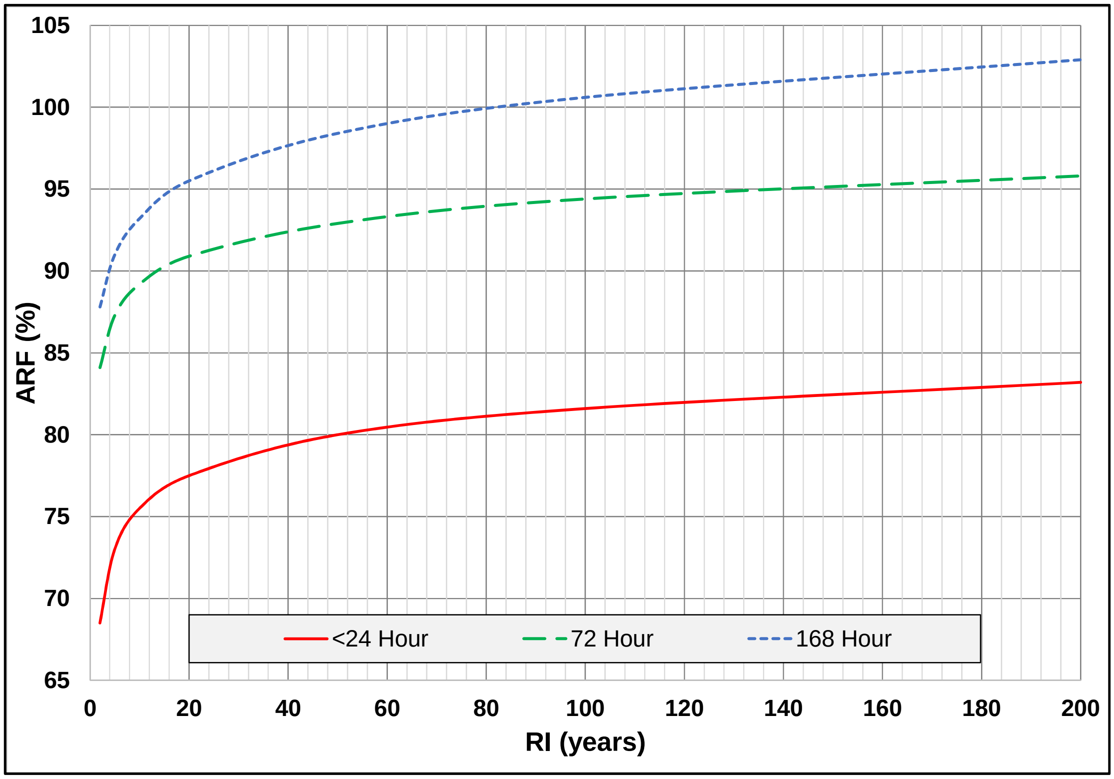

Figure B.21: Geographically-centred ARFs with corresponding RIs in QC C51C

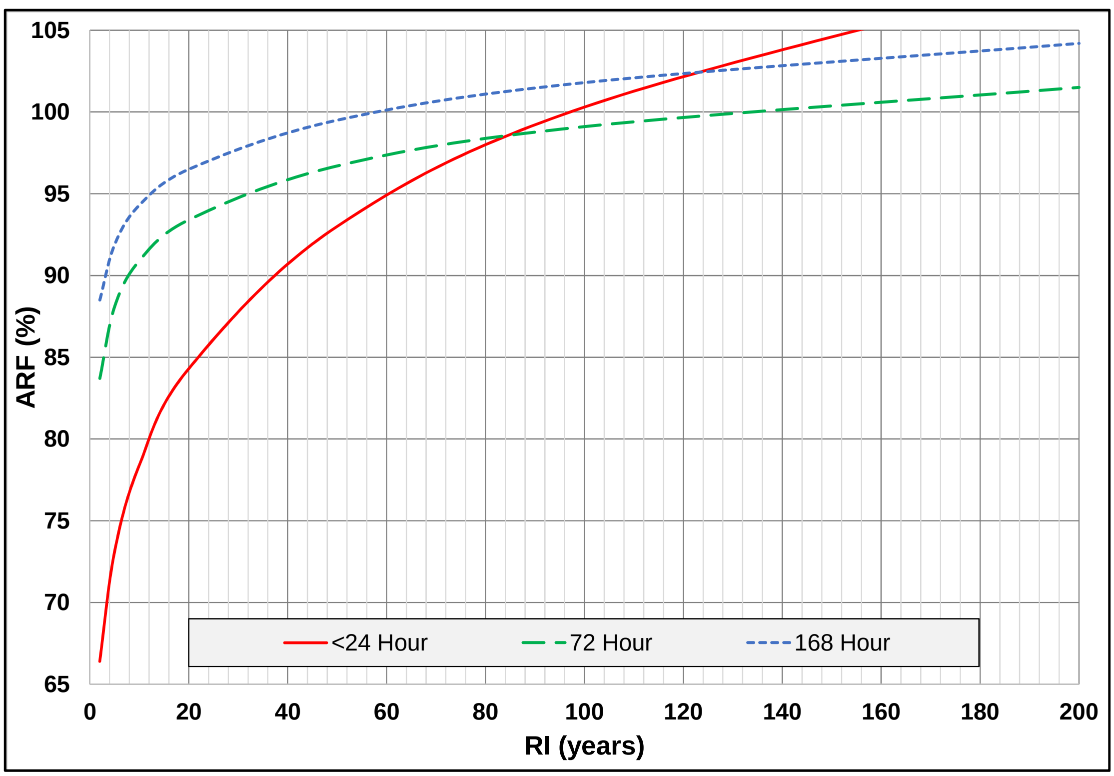

Figure B.22: Geographically-centred ARFs with corresponding RIs in QC C51D

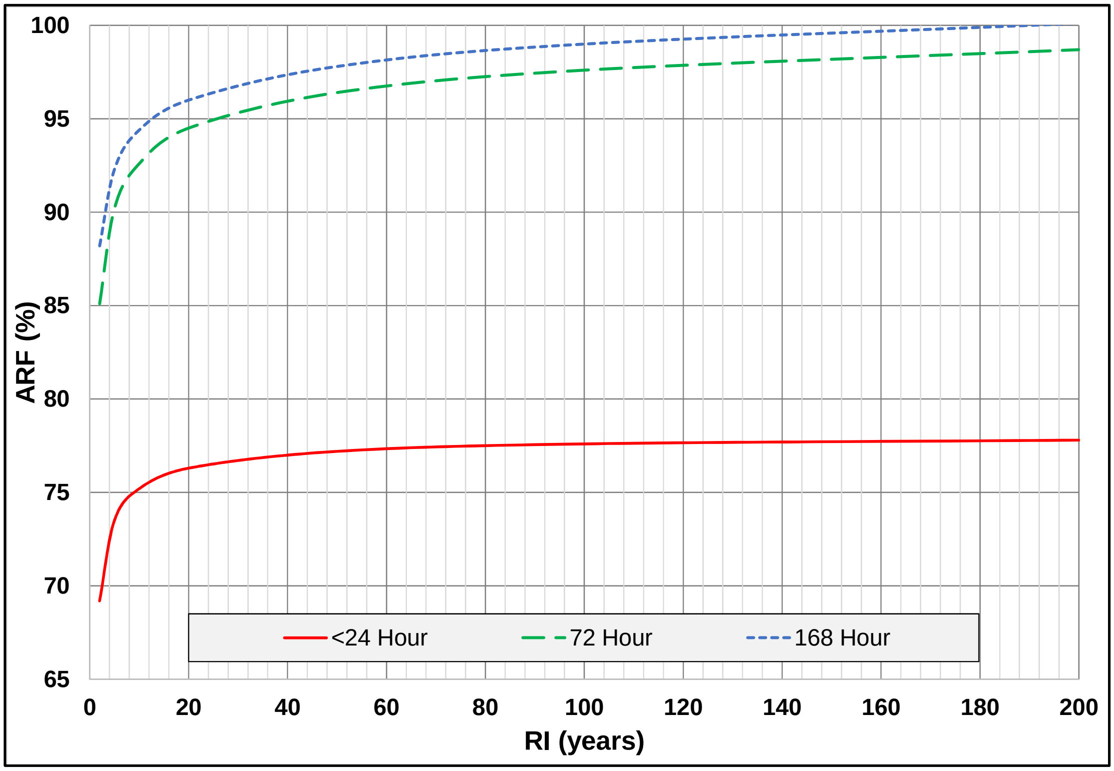

Figure B.23: Geographically-centred ARFs with corresponding RIs in QC C51E

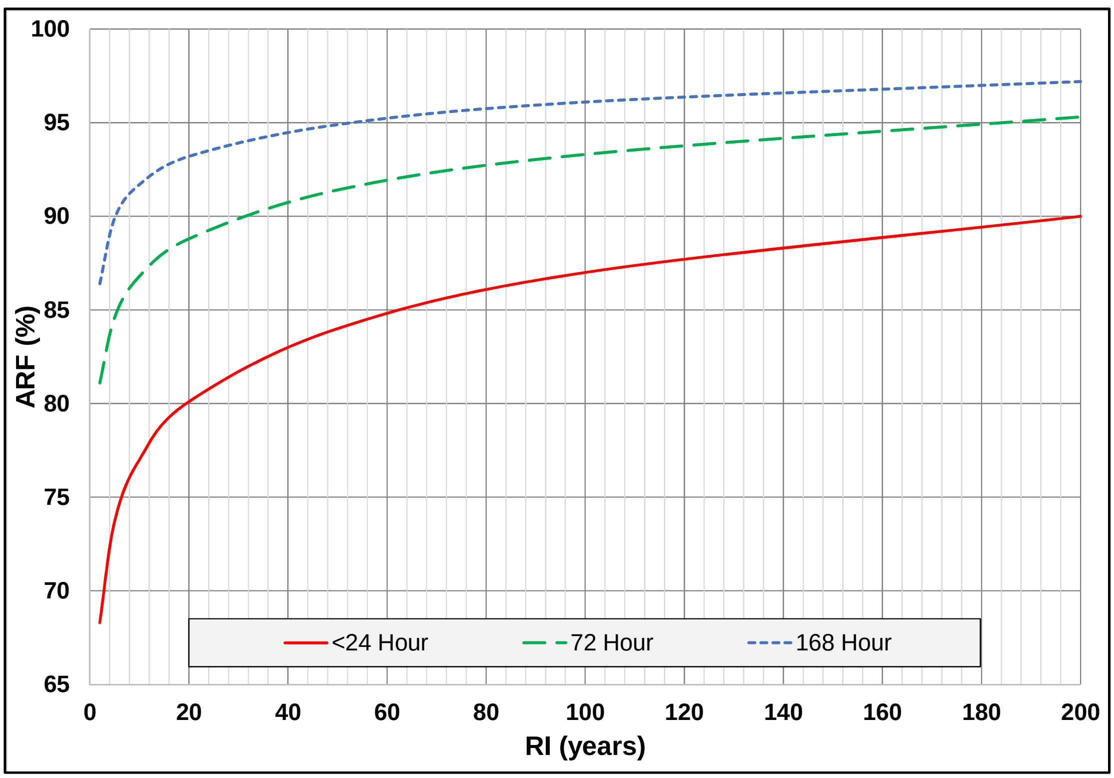

Figure B.24: Geographically-centred ARFs with corresponding RIs in QC C51F

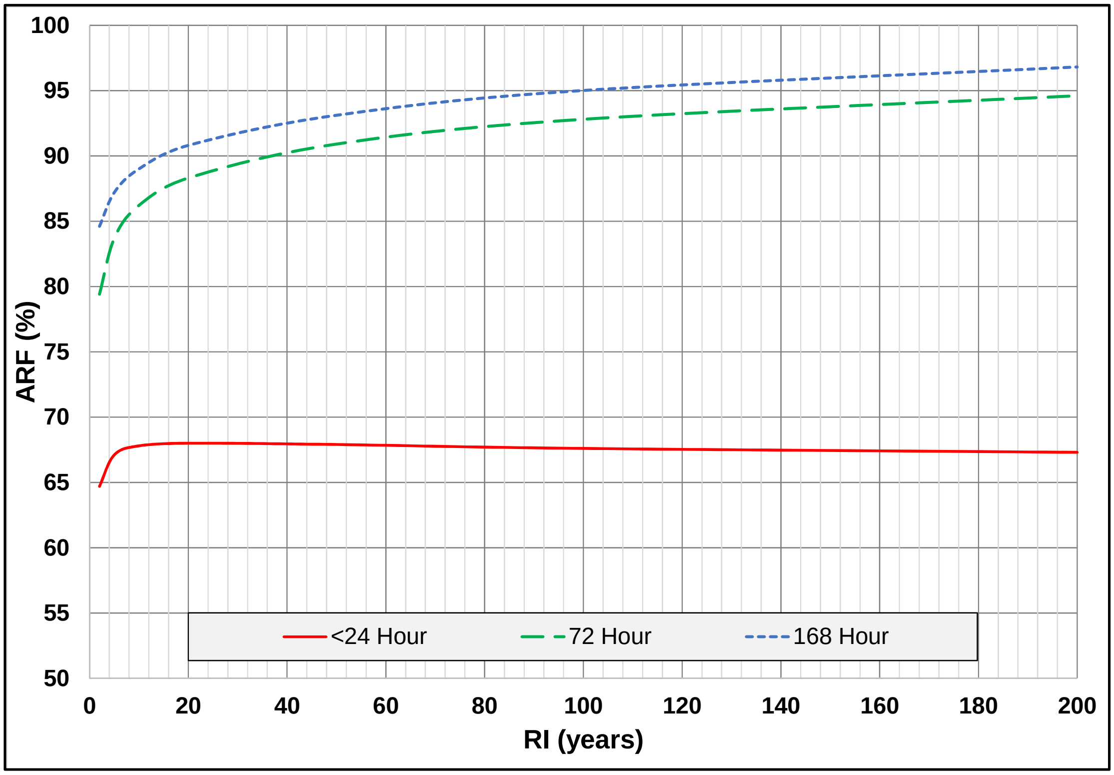

Figure B.25: Geographically-centred ARFs with corresponding RIs in QC C51G

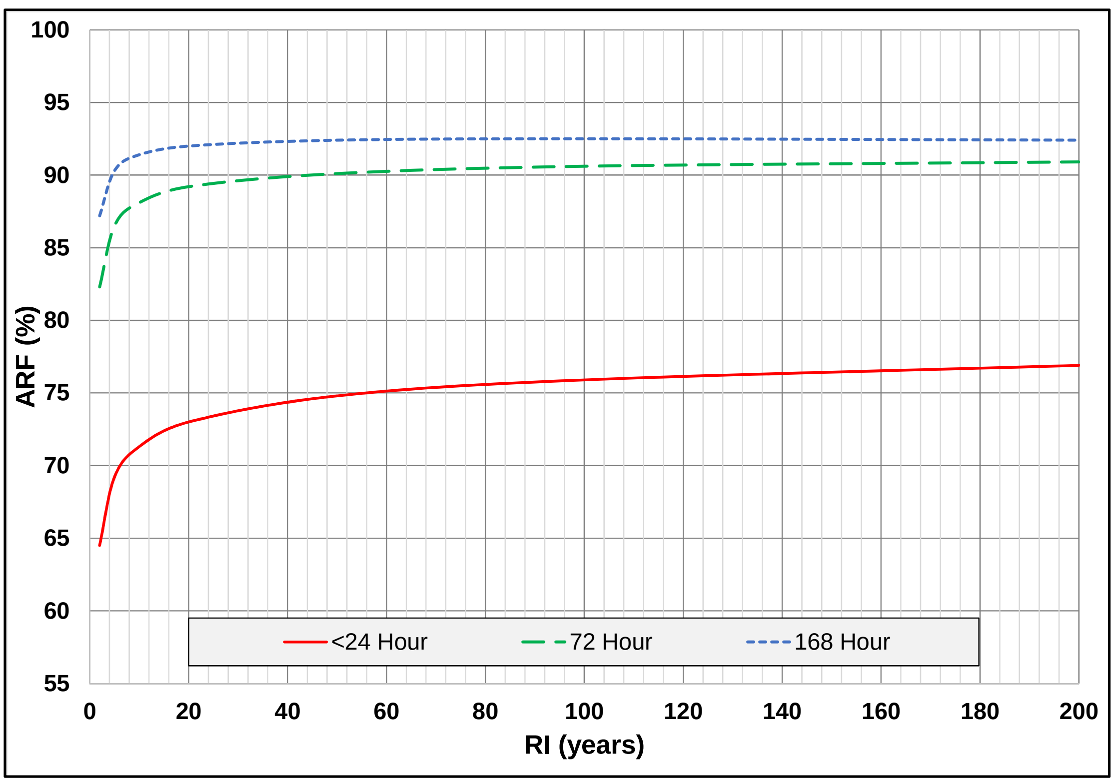

Figure B.26: Geographically-centred ARFs with corresponding RIs in QC C51H

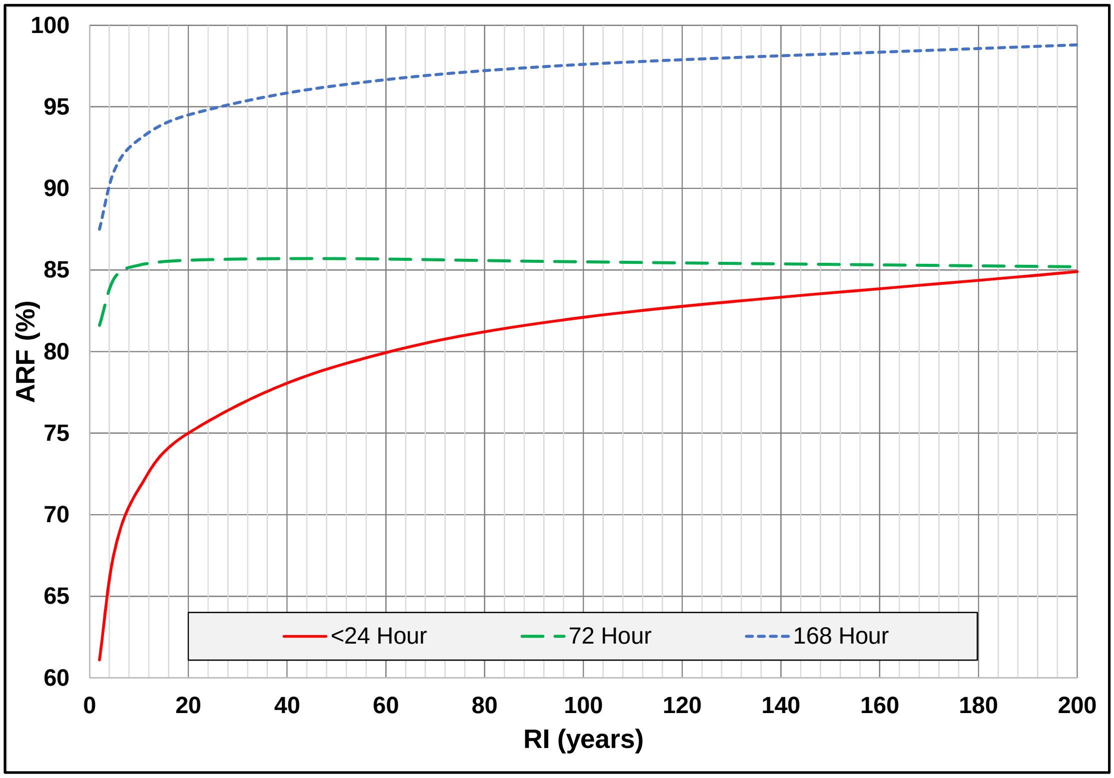

Figure B.27: Geographically-centred ARFs with corresponding RIs in QC C51J

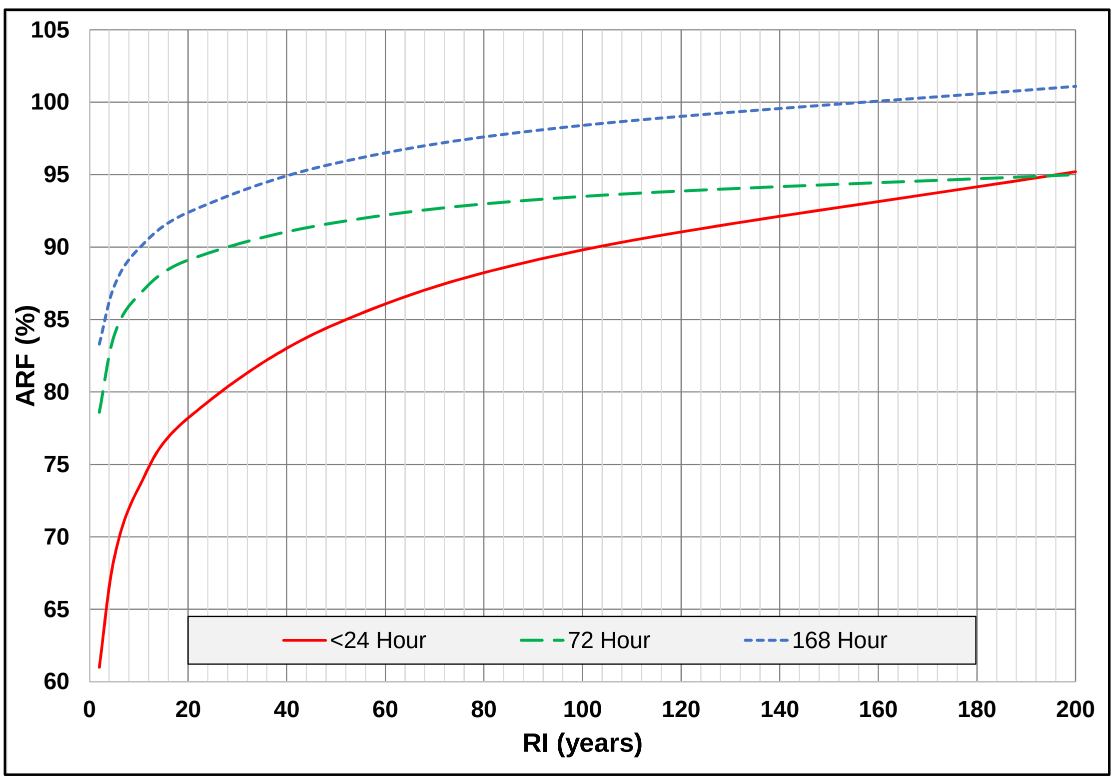

Figure B.28: Geographically-centred ARFs with corresponding RIs in QC C51K

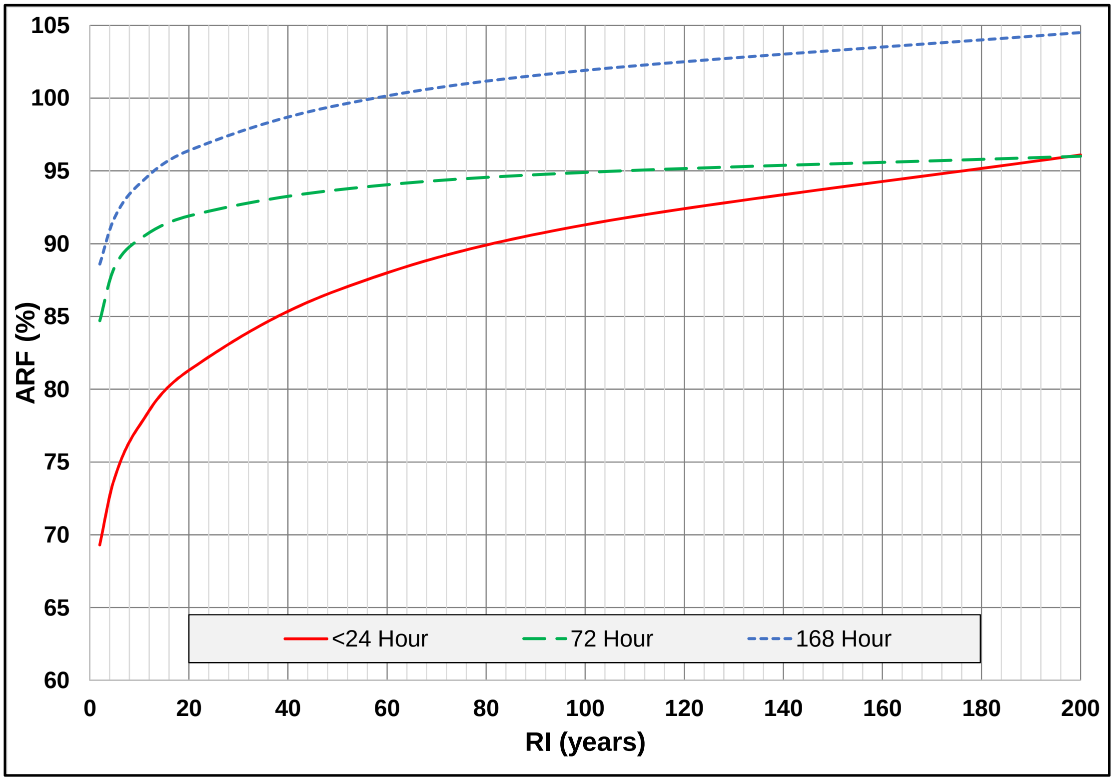

Figure B.29: Geographically-centred ARFs with corresponding RIs in QC C51L

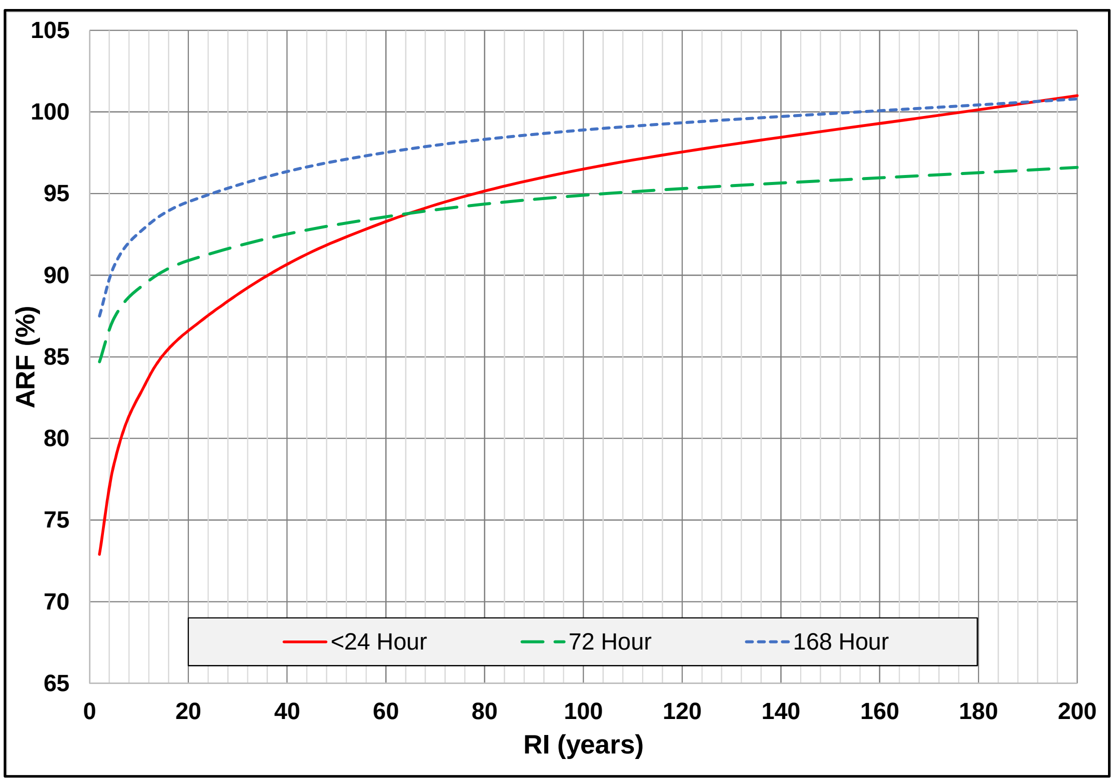

Figure B.30: Geographically-centred ARFs with corresponding RIs in QC C51M

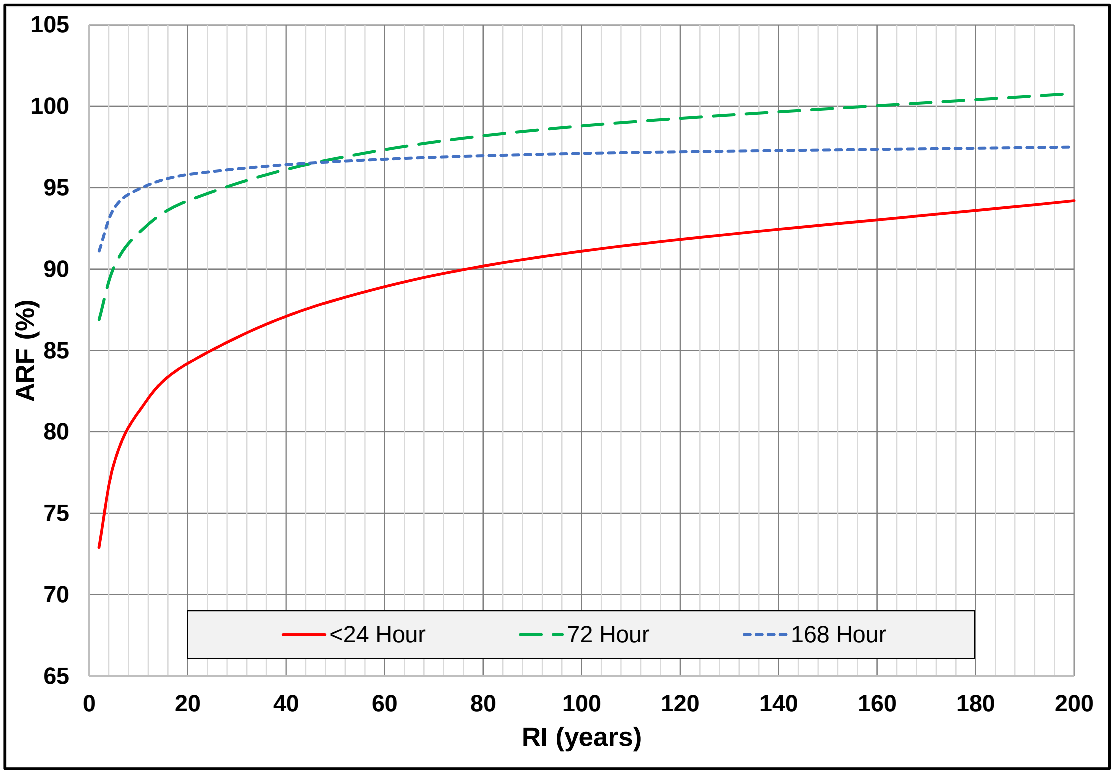

Figure B.31: Geographically-centred ARFs with corresponding RIs in QC C52A

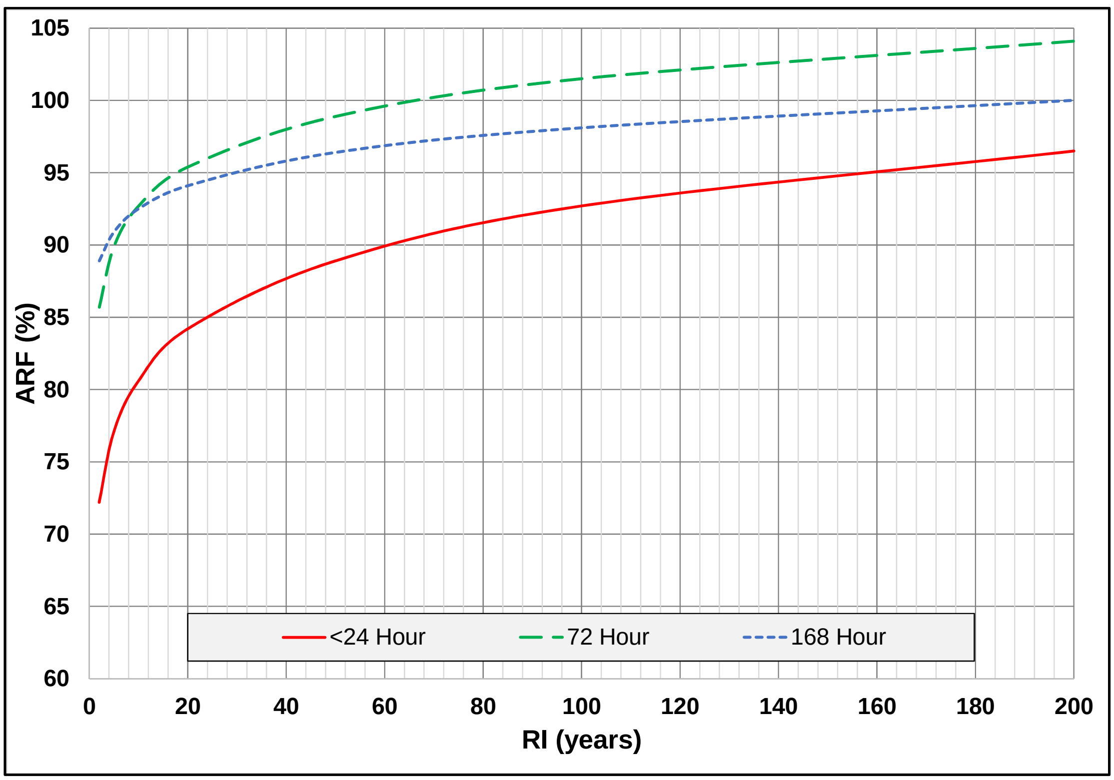

Figure B.32: Geographically-centred ARFs with corresponding RIs in QC C52B

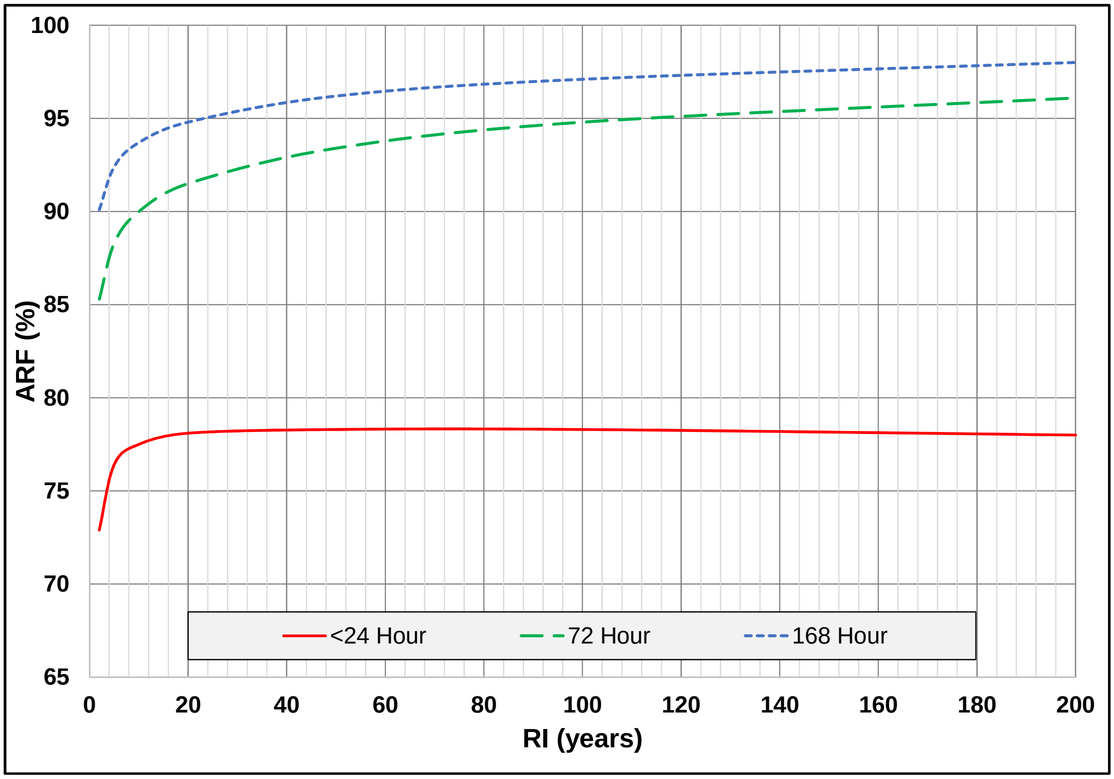

Figure B.33: Geographically-centred ARFs with corresponding RIs in QC C52C

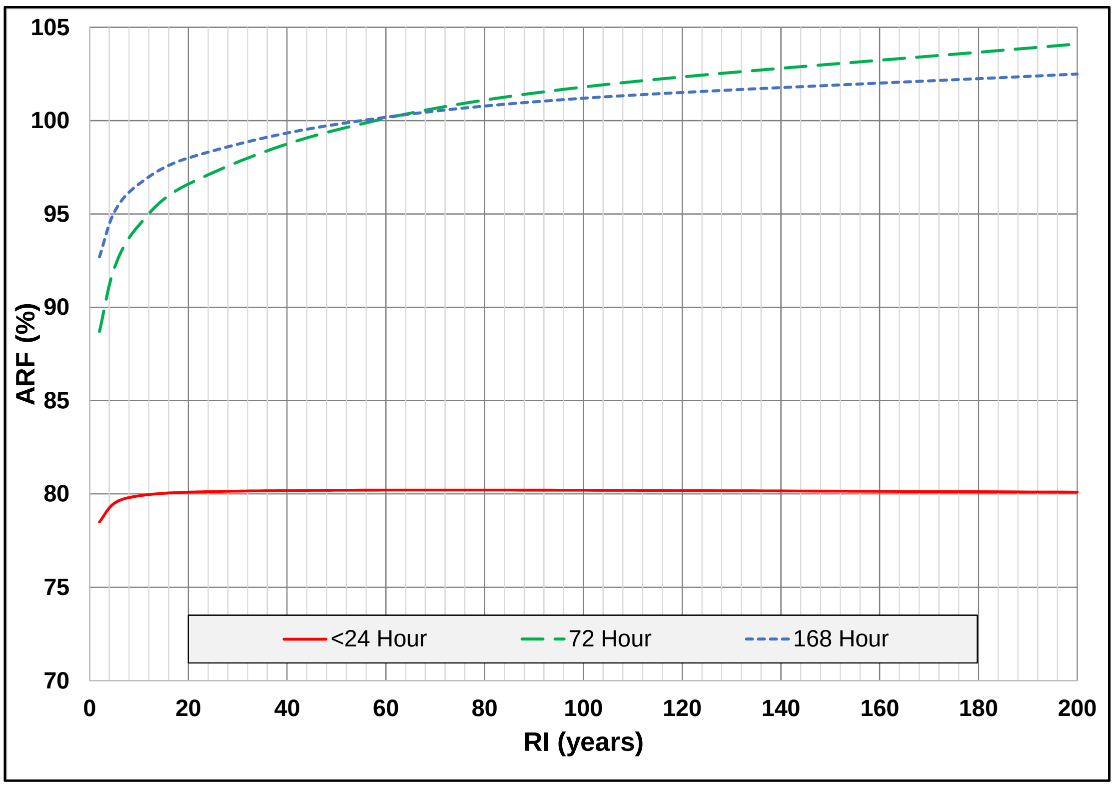

Figure B.34: Geographically-centred ARFs with corresponding RIs in QC C52D

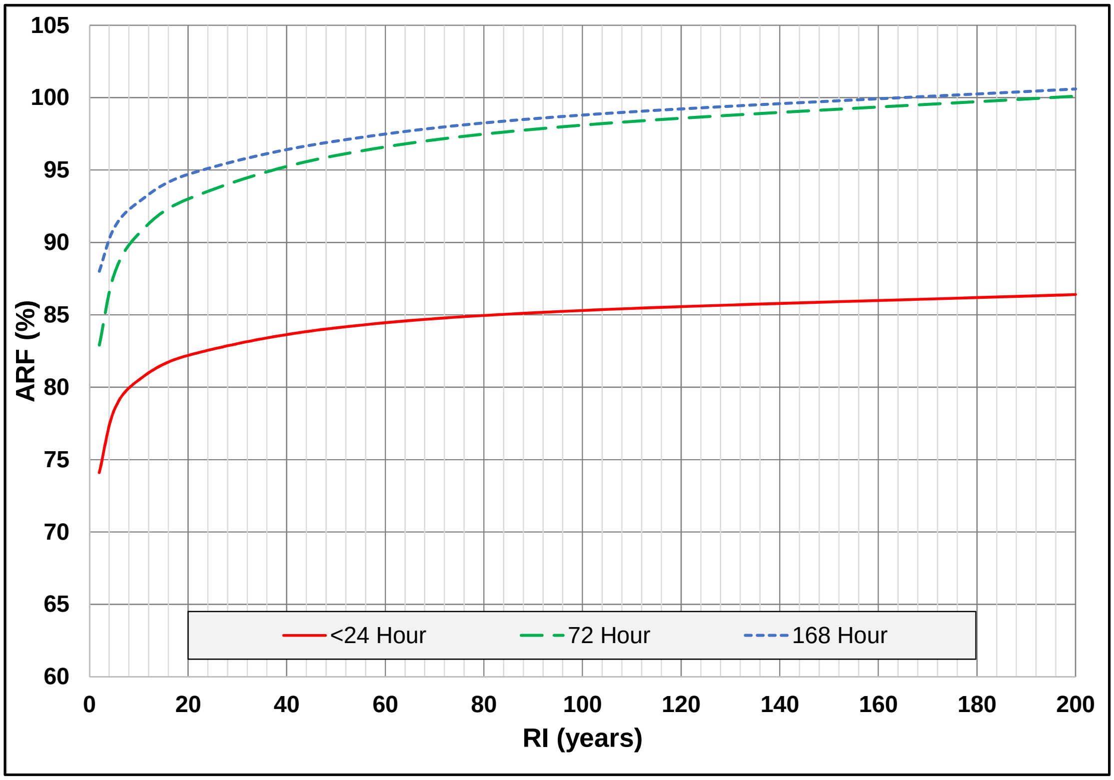

Figure B.35: Geographically-centred ARFs with corresponding RIs in QC C52E

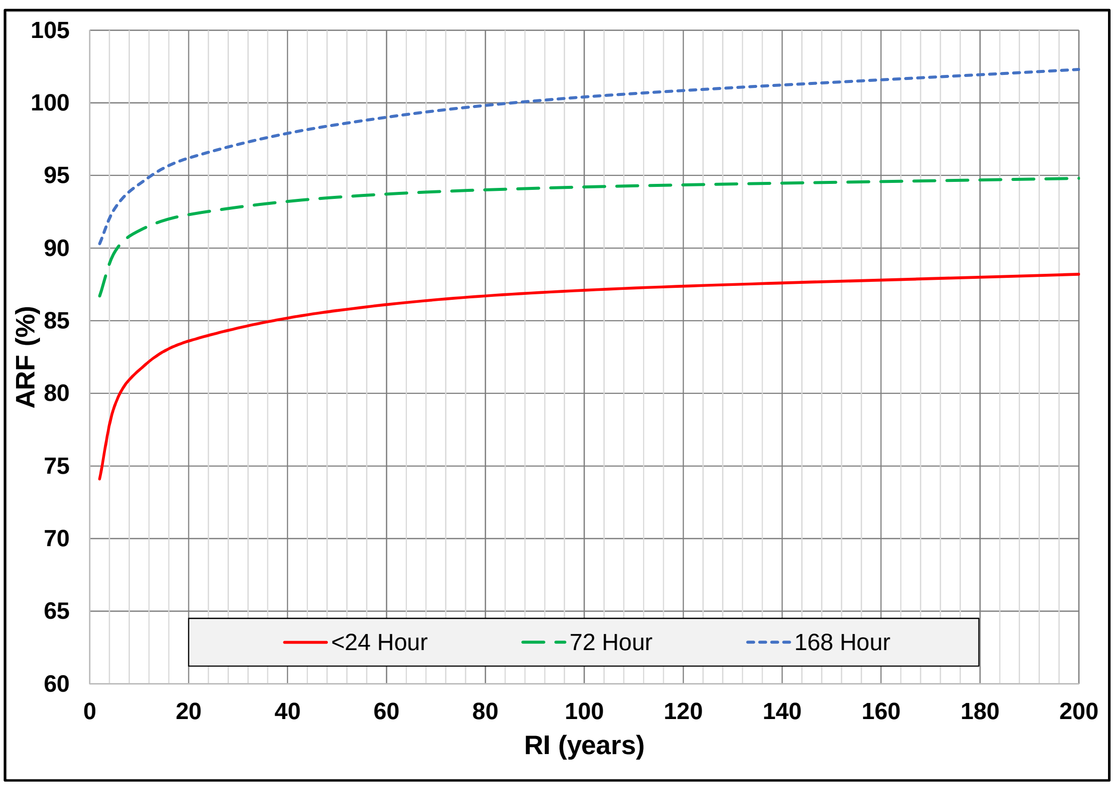

Figure B.36: Geographically-centred ARFs with corresponding RIs in QC C52F

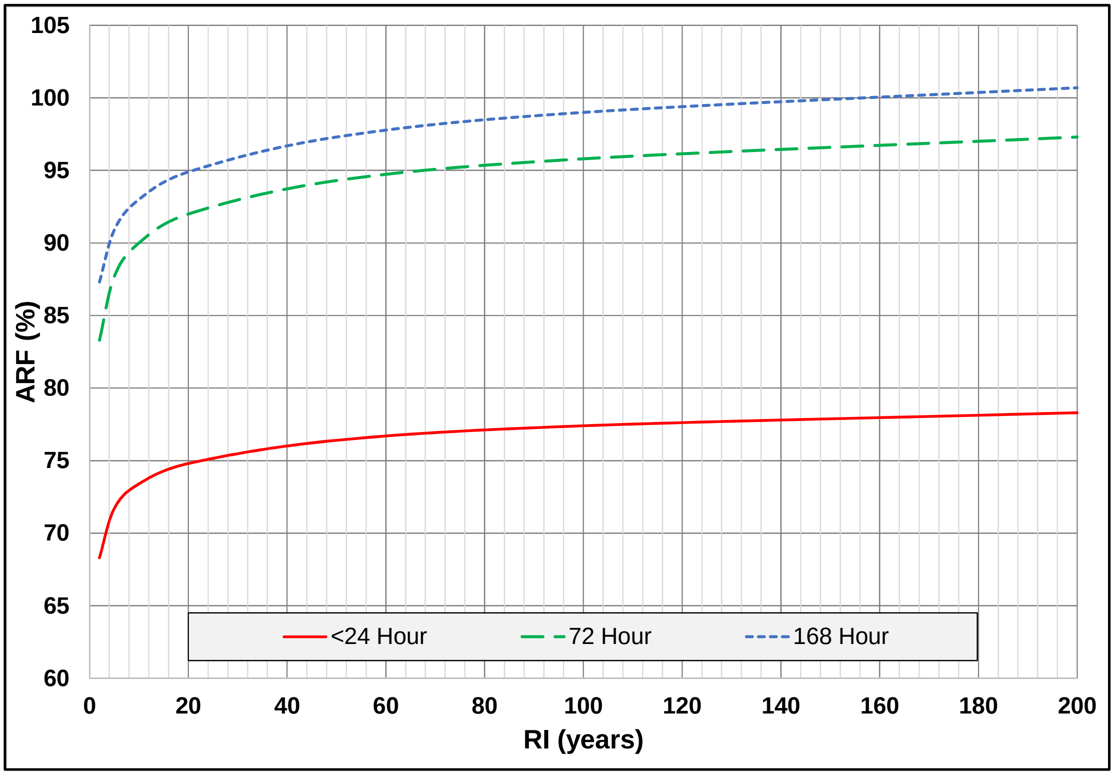

Figure B.37: Geographically-centred ARFs with corresponding RIs in QC C52G

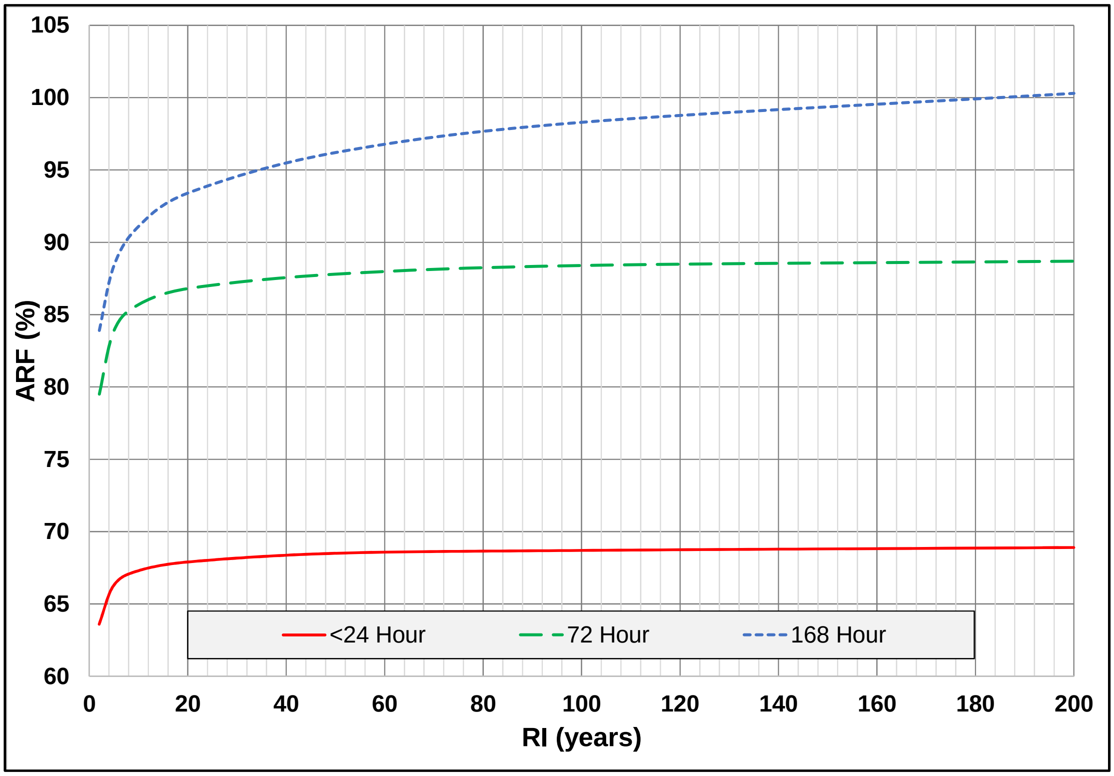

Figure B.38: Geographically-centred ARFs with corresponding RIs in QC C52H

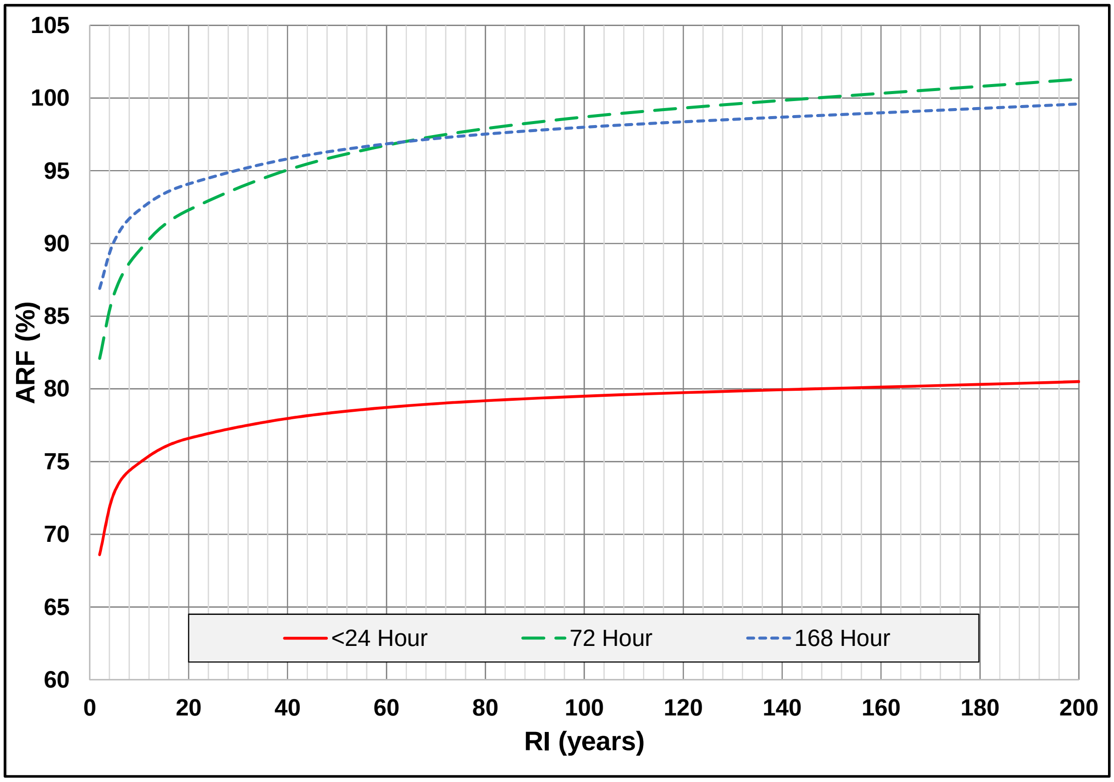

Figure B.39: Geographically-centred ARFs with corresponding RIs in QC C52J

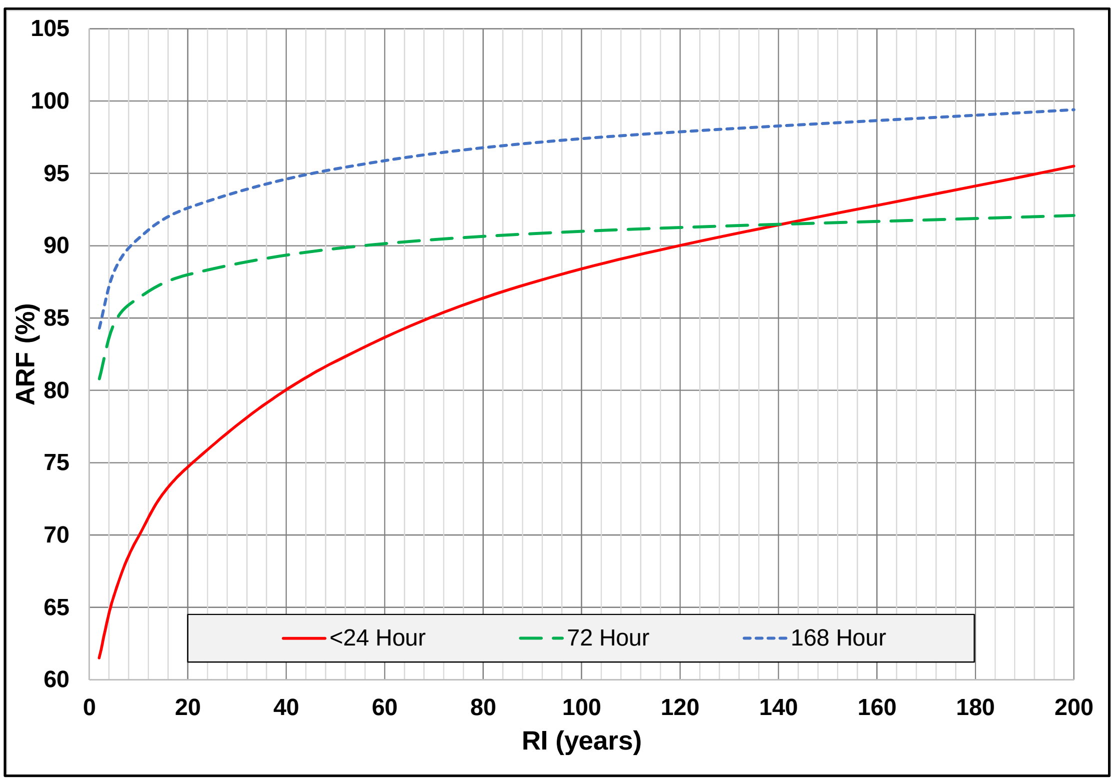

Figure B.40: Geographically-centred ARFs with corresponding RIs in QC C52K

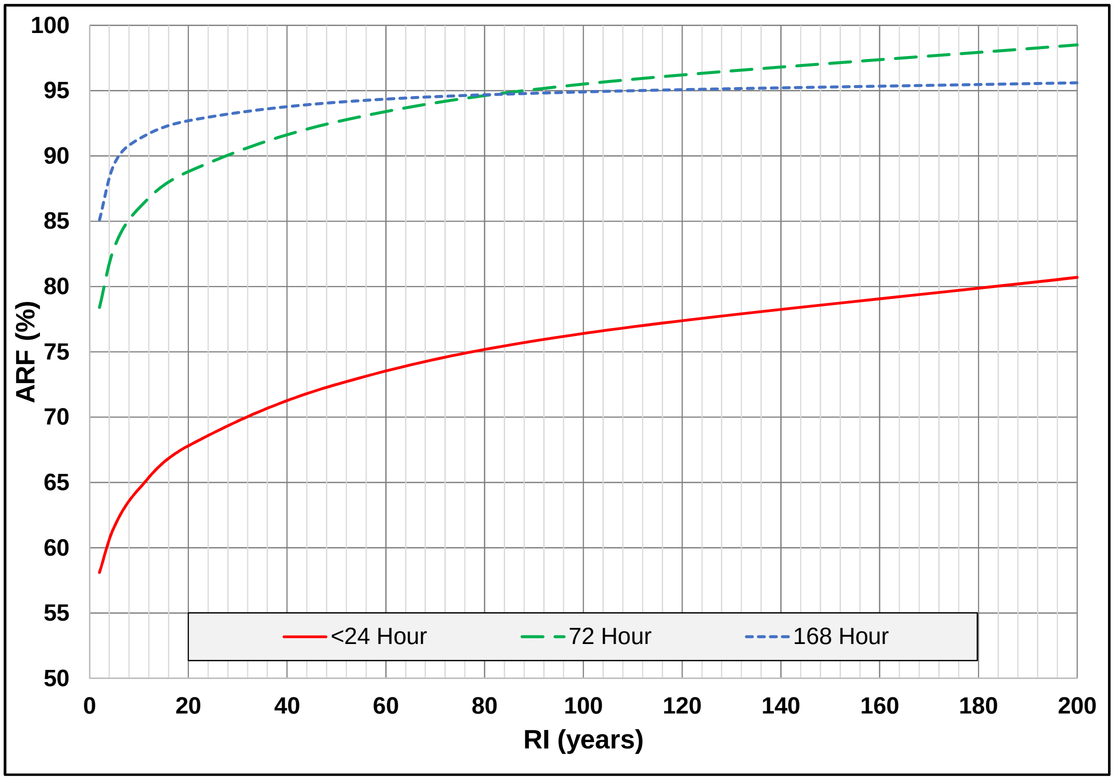

Figure B.41: Geographically-centred ARFs with corresponding RIs in QC C52L